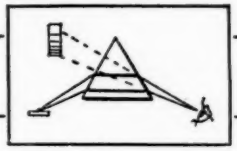
Journal of the OPTICAL SOCIETY *of* AMERICA

VOLUME 26
No. 5



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Published monthly
at Prince and Lemon Sts., Lancaster, Pa.
by the
AMERICAN INSTITUTE OF PHYSICS
Incorporated
175 Fifth Avenue, New York, N. Y.
FOR AND UNDER THE EDITORIAL SUPERVISION OF
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Journal of the
OPTICAL SOCIETY
of AMERICA

VOLUME 26 MAY, 1936 NUMBER 5

A New Theory of Optical Image Formation*

M. HERZBERGER, *Eastman Kodak Research Laboratories, Rochester, New York*

(Received February 22, 1936)

A report including the generalization of Gaussian theory of image formation with regard to an arbitrary ray; the location of points with a sharp image, and points whose image caustics are symmetrical; the derivation of the angle eiconal W for systems with certain special qualities; and a discussion of the qualities of ideal optical systems.

NOTWITHSTANDING all protests from optical writers, it is still a general belief that the image formation in optical systems is a one-to-one point correspondence, according to the laws of collinear geometry. Most physicists know little more of optics than these laws. If we except for the moment one very special case (that having the qualities of the plane mirror), collinear geometry gives a line-to-line correspondence, contradicting optical laws. The reasons why these false ideas are so prevalent are, first, that these laws are approximately true for the rays in the neighborhood of the axis of a rotation-symmetrical optical system, which includes practically all optical instruments; and, second, they are simple laws, and, beside the law of Malus, there are very few simple laws known which are valid for a general ray and its neighborhood.

In the first part of this paper (a continuation of the work carried out by the author and H. Boegehold in Germany since 1928), the author will show how the laws of Gaussian optics can be derived as special cases of generally valid laws, which we shall now discuss.

We have a given rotationally symmetrical optical system, and an arbitrary point A off the axis. This point and the axis determine a plane, which is the meridian plane of the object point. Now we consider an arbitrary ray through A which does not lie in the meridian plane. The corresponding image ray will intersect the meridian plane at a point A', which we shall call the *diapoint* of A. Of course, if we choose another ray through A, we get another diapoint, so that, in general, we have for each object point a two-dimensional manifold of diapoints. If the manifold is only one-dimensional for a certain point, that is, if the diapoints form a curve, we can prove that the image caustic is rotation-symmetrical around this curve. If there is only a single diapoint, A has a sharp image, and the diapoint is the singular image point.

Now that we have seen the importance of the diapoints in the discussion of the image formation, let us consider an arbitrary ray. To each of its points corresponds a certain diapoint. We drop the perpendiculars from point and diapoint to the axis, and call the ratio β' of these perpendiculars the *diamagnification*. Now we can show that the change of diamagnification when object or image point is shifted along the ray follows

* Communication No. 584.

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the same laws that are valid in Gaussian optics.

The second section will deal with the image formation for infinite objects, and with the qualities of the ideal photographic objective. In the third section, we shall investigate the image of finite points, introducing a new eiconal to facilitate the calculation. The last section will give examples illustrating how the methods found in this paper may be applied to the solution of special problems.

Section I. Gaussian Optics for Skew Rays

We set up in object and image space a coordinate system, such that the origins are arbitrary, but the z and z' axes coincide with the axis of the rotation-symmetrical optical instrument. The manifold of rays in space is four-dimensional, but according to the rotation-symmetry, if we have one ray and its image ray given, and we rotate the object ray around the axis, the image ray will rotate simultaneously. Accordingly, we need only three parameters, and may take

$$\kappa=\cos\sigma,\quad \lambda=\cos\vartheta,\quad \mu=\cos\sigma', \qquad (\text{I, } 1)$$

where σ is the angle between ray and axis in the object space, σ' the corresponding angle in the image space, and ϑ is the angle between object and image rays. All three angles remain constant as the object ray with its corresponding image ray is rotated around the axis. If object and image ray are given in our coordinate systems by the direction cosines ξ, η, ζ and ξ', η', ζ', respectively, we have

$$\begin{aligned}\kappa&=(1-(\xi^2+\eta^2))^{\frac{1}{2}}=(1-a/n^2)^{\frac{1}{2}},\\ \lambda&=\xi\xi'+\eta\eta'+\zeta\zeta'=b/2nn'\\ &\qquad+((1-a/r^2)(1-c/n'^2))^{\frac{1}{2}},\\ \mu&=(1-(\xi'^2+\eta^2))^{\frac{1}{2}}=(1-c/n'^2)^{\frac{1}{2}},\end{aligned} \qquad (\text{I, } 2)$$

if we introduce the usual abbreviations

$$\begin{aligned}a&=n^2(\xi^2+\eta^2),\quad b=2nn'(\xi\xi'+\eta\eta'),\\ c&=n'^2(\xi'^2+\eta'^2).\end{aligned} \qquad (\text{I, } 3)$$

In most textbooks, as well as in the previous publications of the author, these quantities a, b, c are taken as the variables, but it will be seen later that the introduction of κ, λ, μ, which the author calls *natural coordinates*, offers great advantages.

If we consider the coordinates x, y, x', y', of the points where object and image ray intersect the coordinate planes $z\,O$, $x'\,O$, respectively, we find from the so-called Bruns equations

$$\begin{aligned}nx&=nW_\xi=2W_a n^2\xi+2W_b nn'\xi'\\ &=-(W_\kappa+\mu W_\lambda)\xi/\kappa+W_\lambda\xi',\\ ny&=nW_\eta=2W_a n^2\eta+2W_b nn'\eta'\\ &=-(W_\kappa+\mu W_\lambda)\eta/\kappa+W_\lambda\eta',\\ n'x'&=-n'W_{\xi'}=-2W_b nn'\xi-2W_c n'^2\xi'\\ &=-W_\lambda\xi+(W_\mu+\kappa W_\lambda)\xi'/\mu,\\ n'y'&=-n'W_{\eta'}=-2W_b nn'\eta-2W_c n'^2\eta'\\ &=-W_\lambda\eta+(W_\mu+\kappa W_\lambda)\eta'/\mu.\end{aligned} \qquad (\text{I, } 4)$$

If we shift the coordinate planes in object and image space the distances z and z', respectively, along the axis (z and z' have yet no connection with each other), we find for the coordinates of the intersection points in these new planes

$$\begin{aligned}nx^*&=-(W_\kappa+\mu W_\lambda-nz)\xi/\kappa+W_\lambda\xi',\\ n'x^{*\prime}&=-W_\lambda\xi+(W_\mu+\kappa W_\lambda-n'z')\xi'/\mu\end{aligned} \qquad (\text{I, } 5)$$

and analogously for y, y'. If we choose z' such that

$$\begin{aligned}\frac{n'x'}{nx}=\frac{n'}{n}\beta'=\mathfrak{B}'&=\frac{\kappa W_\lambda}{W_\kappa+\mu W_\lambda-nz}\\ &=\frac{W_\mu+\kappa W_\lambda-n'z'}{\mu W_\lambda}\end{aligned} \qquad (\text{I, } 6)$$

(and notice that the value of this fraction is the reduced diamagnification $\mathfrak{B}'$), we have a coordination of the points on object and image ray such that the two coordinated points are in the same meridian plane. This means that (I, 6) gives us the coordination between the object point and its diapoint.

Let us find the diafocal point. If we set $z=\infty$,

$$n'z'=W_\mu+\kappa W_\lambda, \qquad (\text{I, } 7)$$

which is the diafocal point of the ray. If we put (7) into (5), we find that

$$n'x_F^{*\prime}=-W_\lambda\xi,\quad n'y_F^{*\prime}=-W_\lambda\eta \qquad (\text{I, } 8)$$

or, if we designate with h' the perpendicular distance from the diafocal point to the axis, we have

$$n'h' = \mathfrak{h}' = W_\lambda \sin\sigma = -\kappa W_\lambda \tan\sigma. \quad \text{(I, 9)}$$

We designate f_D' the diafocal length, and define it

$$\mathfrak{f}_D' = -\kappa W_\lambda \quad \text{(I, 10)}$$

and, in the same way, we find $\mathfrak{f}_D = +\mu W_\lambda$. The focal lengths fulfill the equation

$$\mathfrak{f}'/\mathfrak{f} = -\kappa/\mu \quad \text{or} \quad \mu\mathfrak{f}' + \kappa\mathfrak{f} = 0 \quad \text{(I, 11)}$$

and with the help of (I, 10) we can write (6)

$$\beta' = \frac{\mathfrak{f}_D'}{nz - (W_\kappa + 2\mu W_\lambda)} = \frac{n'z' + (W_\mu + 2\kappa W_\lambda)}{\mathfrak{f}_D} \quad \text{(I, 12)}$$

or

$$\beta' = -\mathfrak{f}'/\mathfrak{z}_F = \mathfrak{z}_F'/\mathfrak{f} \quad \text{(I, 13)}$$

if $\mathfrak{z}_F$ is the optical distance from the object diafocal point to the object point, and $\mathfrak{z}_F'$ the same distance in the image space. These laws are the same as the laws of Newton, valid within the limits of Gaussian optics. The only difference is that for paraxial rays

$$\cos\sigma = \kappa \to 1, \quad \cos\sigma' = \mu \to 1$$

so that we can state for this special case

$$\mathfrak{f} + \mathfrak{f}' = 0. \quad \text{(I, 14)}$$

We remember that we have defined the diapoint only for skew rays. However, the algebraic definition given in (I, 6) still holds if we consider a ray lying in the meridian plane, and holds even when the object point, and therefore the image point, is situated on the axis of the system. Therefore, in the future we shall define the diapoint and the diamagnification algebraically by Eq. (I, 6).

For the perpendiculars h and h' from object and diapoint to the axis, we find from (5) and (6)

$$\mathfrak{h}^2 = \left(\frac{W_\kappa + \mu W_\lambda - \mathfrak{z}}{\kappa}\right)^2 (1-\kappa^2) - 2W_\lambda \frac{W_\kappa + \mu W_\lambda - \mathfrak{z}}{\kappa}(\lambda - \kappa\mu) + W_\lambda^2(1-\mu^2) \quad \text{(I, 15)}$$

$$= \frac{W_\lambda^2}{\beta'^2}[\beta'^2 + 1 - \lambda\beta' - (\beta'\mu - \kappa)^2],$$

$$\mathfrak{h}'^2 = W_\lambda^2[\beta'^2 + 1 - \lambda\beta' - (\beta'\mu - \kappa)^2] = \beta'^2\mathfrak{h}^2.$$

Instead of investigating the diapoints of a certain fixed point, given by h, z, we can ask also for the distribution of the diamagnification. On each ray we can find a single pair of object and image points with a given value of β'. That means that in general there is a four-dimensional manifold of object and image points. The case is interesting when the manifolds in object and image space are only two-dimensional. These two-dimensional manifolds determine then object and image surfaces which are not only sharp images of each other, but also, according to an expression introduced in the author's *Strahlenoptik*, *true* images, having the lateral magnification constant for all points of the surfaces. We have true images of the infinite planes ($\beta' = 0$, ∞, respectively), if $2\mu W_\lambda = \mathfrak{f}$ or $2\kappa W_\lambda = -\mathfrak{f}'$ is constant. This particular case will be considered specially.

Section II. Diapoints of the Points of the Infinite Plane; Photographic Objective, Ocular, Janus Objective

Let us first investigate the diapoints belonging to a point of the infinite object plane.

We have

$$\mathfrak{z}' = n'z' = -(W_\mu + \kappa W_\lambda),$$
$$\mathfrak{h}' = n'h' = -W_\lambda(1-\kappa^2)^{\frac{1}{2}}. \quad \text{(II, 1)}$$

In these equations we have an arbitrary point of the infinite plane given by a constant value of κ.

In (II, 1), z' and h' for a certain value $\kappa = \kappa_0$ are functions of two variables, μ and λ. Eqs. (II, 1), therefore, give in general a two-dimensional manifold of diapoints. Let us ask, then, for the condition that this manifold is one-dimensional. We have in general

$$d(n'z') = -(W_{\mu\lambda} + \kappa W_{\lambda\lambda})d\lambda - (W_{\mu\mu} + \kappa W_{\mu\lambda})d\mu,$$
$$d(n'h') = -W_{\lambda\lambda}(1-\kappa^2)^2 d\lambda - W_{\mu\lambda}(1-\kappa^2)\, d\mu. \quad \text{(II, 2)}$$

We can say that this manifold is one-dimensional only if

$$W_{\lambda\lambda}W_{\mu\mu} - W_{\lambda\mu}^2 = 0 \quad \text{(II, 3)}$$

for our special value of κ, or, that for this value of κ,

$$W_\mu = f(W_\lambda). \quad \text{(II, 4)}$$

In this case, it is possible to eliminate λ or μ from (II, 1), and we get the equation of the curve around which the rays are symmetrical.

Let us consider a special case; that when the caustic is symmetrical about a straight line.

Then we must have

$$W_\mu+\kappa W_\lambda=A\,W_\lambda(1-\kappa^2)^{\frac{1}{2}}+B. \quad \text{(II, 5)}$$

That means that for our special value of κ

$$W=B\mu+f(\lambda+\mu[A(1-\kappa^2)^{\frac{1}{2}}-\kappa]). \quad \text{(II, 6)}$$

The infinite point has a sharp image only if

$$W_{\lambda\lambda}=W_{\lambda\mu}=W_{\mu\mu}=0, \quad \text{(II, 7)}$$

which means that, for our particular value of κ, W_λ and W_μ are constant.

Now we ask for the condition that all points of the infinite plane have rotation-symmetrical images. Then (II, 3) must be fulfilled for all values of κ, and we get

$$W_\mu=f(\kappa,\ W_\lambda). \quad \text{(II, 8)}$$

The derivative W_μ is an arbitrary function of κ and W_λ. Especially are all image caustics rotation-symmetrical in a narrower sense, that is, around straight lines, if (II, 5) is fulfilled for all values of κ, where, of course, A and B may be functions of κ. That leads to

$$W=f(\kappa)\mu+g(\lambda+\mu h(\kappa)), \quad \text{(II, 9)}$$

where f, g, h, are arbitrary functions.

The straight line for a particular value of κ, when (II, 9) is fulfilled, is indicated by

$$n'z'=\frac{h(\kappa)+\kappa}{(1-\kappa^2)^{\frac{1}{2}}}n'h'-f(\kappa). \quad \text{(II, 10)}$$

These lines pass through a common point off the axis only if $f(k)$ is constant.

We ask now for the condition that all points of the infinite plane have a sharp image. Then (II, 7) must be fulfilled for all values of κ. That gives

$$W_\lambda=f(\kappa), \quad W_\mu=g(\kappa),$$

or

$$W=f(\kappa)\lambda+g(\kappa)\mu. \quad \text{(II, 11)}$$

If (II, 11) is fulfilled, we can find the image surface by eliminating κ from the equations

$$n'z'=-[g(\kappa)+\kappa f(\kappa)];\ n'h'=f(\kappa)(1-\kappa^2)^{\frac{1}{2}}. \quad \text{(II, 12)}$$

The image is a plane only if

$$g(\kappa)+\kappa f(\kappa)=0, \quad \text{or} \quad W=f(\kappa)(\lambda-\kappa\mu). \quad \text{(II, 13)}$$

In this case, and only in this case do we have a sharp plane image of the infinite plane. But this image can still have distortion. We measure the distortion as the amount of $n'h'/n\tan\sigma$, which is according to (II, 12)

$$n'h'/n\tan\sigma=\kappa f(\kappa). \quad \text{(II, 14)}$$

The image is free of distortion only if this quantity is constant. Then we have $f(\kappa)=C/\kappa$,

$$W=(C/\kappa)(\lambda-\kappa\mu)=C\lambda/\kappa-C\mu. \quad \text{(II, 15)}$$

Dropping the linear members of κ and μ means, as we have seen, only a shift of the origin. Therefore we can say that

$$W=C\lambda/\kappa \quad \text{(II, 16)}$$

is the eiconal for the ideal photographic objective.

In the same way, we can deal with the image formation of the infinite image plane. That means that we can deal with the ocular system.

For the ocular system, we need only interchange κ and μ. The diapoints are given by

$$nz=W_\kappa+\mu W_\lambda, \quad nh=W_\lambda(1-\mu^2)^{\frac{1}{2}}. \quad \text{(II, 17)}$$

An infinite point has a symmetrical image caustic if, for this value $\mu=\mu_0$,

$$W_\kappa=f(W_\lambda) \quad \text{(II, 18)}$$

and, in particular, the axis of symmetry is a straight line if

$$W_\kappa+\mu_0 W_\lambda=A\,W_\lambda(1-\mu_0^2)^{\frac{1}{2}}+B. \quad \text{(II, 19)}$$

The infinite point has a sharp image if, for this value of μ

$$W_\kappa=C_1, \quad W_\lambda=C_2. \quad \text{(II, 20)}$$

All points of the infinite plane have rotation-symmetrical image caustics if

$$W_\kappa=f(\mu,\ W_\lambda). \quad \text{(II, 21)}$$

The axes of symmetry are straight if

$$W=f(\mu)\kappa+g(\lambda+\kappa h(\mu)) \quad \text{(II, 22)}$$

and

$$nz=\frac{h(\mu)+\mu}{(1-\mu^2)^{\frac{1}{2}}}nh+f(\mu).$$

All points have a sharp image if

$$W=f(\mu)\lambda+g(\mu)\kappa. \quad \text{(II, 23)}$$

The object surface is obtained by eliminating μ from

$$nz=g(\mu)+\mu f(\mu), \quad nh=f(\mu)(1-\mu^2)^{\frac{1}{2}}. \quad \text{(II, 24)}$$

The object is plane only if

$$g(\mu)+\mu f(\mu)=0, \quad W=f(\mu)(\lambda-\mu\kappa), \quad \text{(II, 25)}$$

and orthoscopic only if

$$\mu f(\mu)=C_1, \quad W=C\lambda/\mu-C\kappa$$

or, after shifting the origin

$$W=C\lambda/\mu. \quad \text{(II, 26)}$$

Now it is interesting to see under what conditions an optical instrument can give an equally good image of both infinite planes. Such an objective, which can be used either as a photographic objective or an ocular from either side, let us call a *Janus* objective, according to a suggestion of C. A. Morrison, of these laboratories.

The Janus objective gives a rotation-symmetrical image caustic for each object point only if

$$W_\mu=f(\kappa, W_\lambda), \quad W_\kappa=g(\mu, W_\lambda). \quad \text{(II, 27)}$$

The rotation axes are all straight lines only if

$$W=C_1\mu\kappa+g(\lambda+C_2\mu\kappa). \quad \text{(II, 28)}$$

The axes are then indicated by

$$n'z'=-(C_2+1)\frac{\kappa}{(1-\kappa^2)^{\frac{1}{2}}}n'h'-C_1\kappa,$$
$$nz=(C_2+1)\frac{\mu}{(1-\mu^2)^{\frac{1}{2}}}nh+C_1\mu. \quad \text{(II, 29)}$$

All points have sharp images if

$$W=C_1\lambda+C_2\mu\kappa. \quad \text{(II, 30)}$$

In this case, we can eliminate κ and μ from (II, 24) and (II, 12), respectively, and get

$$n'z'=-(C_1+C_2)\kappa, \quad nz=(C_1+C_2)\mu,$$
$$n'h'=C_1(1-\kappa^2)^{\frac{1}{2}}, \quad nh=C_1(1-\mu^2)^{\frac{1}{2}}, \quad \text{(II, 31)}$$

or

$$(n'h'/C_1)^2+(n'z')^2/(C_1+C_2)^2=1,$$
$$(nh/C_1)^2+(nz/C_1+C_2)^2=1. \quad \text{(II, 32)}$$

Both surfaces are similar ellipsoids which degenerate into planes if $C_1+C_2=0$

$$W=C_1(\lambda-\mu\kappa). \quad \text{(II, 33)}$$

Eq. (II, 33) shows us that there is a possibility of giving a sharp plane image of both infinite planes. But this image is not free of distortion. (II, 14) gives

$$n'h'/n\tan\sigma=-C_1\kappa, \quad nh/n'\tan\sigma'=C_1\mu. \quad \text{(II, 34)}$$

It might be of interest to note that this image formation would give images of the infinite planes within circles, of the radii $(n/n')C_1$ and $(n'/n)C_1$, respectively.

It is impossible to design a Janus objective free of distortion.

Section III. Image Formation for Finite Object Points

In order to give the image formation for finite object points in the same simple way, we use a trick.

The coordinates κ, λ, μ, determine the position of an object and image ray, if we disregard the possible rotation of both rays around the axis. If we give a value of β on object and image ray, a point and its diapoint are determined. Therefore, the values z, h, z', h', may be regarded as functions of these four variables. The eiconal W depends of course on three of these four variables. Now we introduce, instead of this set of variables, a new set ρ, σ, τ, θ, related to κ, λ, μ, β by the equations

$$\rho=\kappa,$$
$$\sigma=\left(\frac{1+\beta^2}{\beta}-2\lambda-\frac{(\mu\beta-\kappa)^2}{\beta}\right)^{\frac{1}{2}},$$
$$\tau=-\mu, \quad \theta=\beta,$$
$$\kappa=\rho, \quad \text{(III, 1)}$$
$$\lambda=\frac{1}{2}\left(\frac{1+\theta^2}{\theta}-\frac{(\tau\theta+\rho)^2}{\theta}-\sigma^2\right)$$
$$\mu=-\tau, \quad \beta=\theta.$$

We introduce now these four new variables into our given eiconal and write

$$W(\kappa, \lambda, \mu)=D(\rho, \sigma, \tau, \theta). \quad \text{(III, 2)}$$

We shall call D the *diapoint eiconal*. The function D depends on four variables. Differentiating, we get from (III, 1)

$$D_\rho = W_\kappa + (\mu\mathfrak{z} - \kappa) W_\lambda / \mathfrak{z},$$

$$D_\sigma = -\sigma W_\lambda = -W_\lambda \left(\frac{1+\mathfrak{z}^2}{\mathfrak{z}} - 2\lambda - \frac{(\mu\mathfrak{z}-\kappa)^2}{\mathfrak{z}} \right)^{\frac{1}{2}},$$

$$D_\tau = -W_\mu + (\mu\mathfrak{z} - \kappa) W_\lambda, \qquad \text{(III, 3)}$$

$$D_\theta = \tfrac{1}{2} W_\lambda [(1-\mu^2) - (1/\mathfrak{z}^2)(1-\kappa^2)].$$

From the last equation we get

$$2\sigma D_\theta + [(1-\tau^2) - (1/\theta^2)(1-\rho^2)] D_\sigma = 0 \qquad \text{(III, 4)}$$

which must be fulfilled for any eiconal.

Next, we introduce (III, 3) into our general equations. Eq. (I, 6) gives

$$\mathfrak{z}' = \frac{\kappa W_\lambda}{W_\kappa + \mu W_\lambda - nz} = \frac{W_\mu + \kappa W_\lambda - n'z'}{\mu W_\lambda},$$

or, resolved according to z and z'

$$\begin{aligned} \mathbf{z} &= nz = W_\kappa + W_\lambda(\mu\mathfrak{z} - \kappa)/\mathfrak{z} = D_\rho, \\ \mathbf{z}' &= n'z' = -W_\mu + W_\lambda(\mu\mathfrak{z} - \kappa) = D_\tau. \end{aligned} \qquad \text{(III, 5)}$$

Eq. (I, 14) gives

$$\mathbf{h} = nh = D_\sigma / \theta^{\frac{1}{2}}, \quad \mathbf{h}' = n'h' = D_\sigma \theta^{\frac{1}{2}} \qquad \text{(III, 6)}$$

so that we get at last the simple equations

$$\begin{aligned} nz &= D_\rho, \qquad nh = D_\sigma / \theta^{\frac{1}{2}}, \\ n'z' &= D_\tau, \quad n'h' = D_\sigma \theta^{\frac{1}{2}}. \end{aligned} \qquad \text{(III, 7)}$$

These equations we will now discuss. We see that if W (and therefore, according to (III, 1), D), is given, we get the coordinates of point and diapoint belonging to a set of coordinates ρ, σ, τ, θ and κ, λ, μ, $\mathfrak{z}$, respectively, simply by differentiating D to its coordinates.

Now we can deal with our problem in the same way as we did in the previous section.

We now investigate a certain point z_0, h_0, to find out whether its diapoints form a two-dimensional manifold, a one-dimensional manifold, or a sharp image. Differentiating (III, 7) gives

$$\begin{aligned} dnz = 0 &= D_{\rho\rho} d\rho + D_{\rho\sigma} d\sigma + D_{\rho\tau} d\tau + D_{\rho\theta} d\theta, \\ dnh = 0 &= \frac{1}{\theta^{\frac{1}{2}}} (F_{\rho\sigma} d\rho + D_{\sigma\sigma} d\sigma + D_{\sigma\tau} d\tau) \\ &\quad + \left(\frac{D_{\sigma\theta}}{\theta^{\frac{1}{2}}} - \frac{1}{2\theta^{\frac{3}{2}}} D_\sigma \right) d\theta, \end{aligned} \qquad \text{(III, 8)}$$

$$dn'z' = D_{\rho\tau} d\rho + D_{\sigma\tau} d\sigma + D_{\tau\tau} d\tau + D_{\tau\theta} d\theta,$$

$$dn'h' = d(\theta nh) = (1/\theta^{\frac{1}{2}}) D_\sigma d\theta.$$

D_σ and D_θ do not vanish, as W_λ may not vanish.

The first equations enable us to eliminate two of the coordinates. Therefore, the manifold of diapoints is, in general, two-dimensional.

We have a rotation-symmetrical caustic only if, for all values of ρ, σ, τ, θ which fulfill the two equations

$$nz_0 = D_\rho, \quad nh_0 = (1/\theta^{\frac{1}{2}}) D_\sigma$$

the condition

$$\begin{vmatrix} D_{\rho\rho} & D_{\rho\sigma} & D_{\rho\tau} \\ D_{\sigma\rho} & D_{\sigma\sigma} & D_{\sigma\tau} \\ D_{\tau\rho} & D_{\tau\sigma} & D_{\tau\tau} \end{vmatrix} = 0 \qquad \text{(III, 9)}$$

is fulfilled.

The axis of symmetry is straight if

$$\theta^{\frac{1}{2}} D_\sigma = A D_\tau + B \qquad \text{(III, 10)}$$

for all values which fulfill (III, 9).

The object point has a sharp image if the matrix (III, 9) is of the first rank.

Then the three equations

$$\begin{vmatrix} D_{\rho\rho} & D_{\rho\sigma} \\ D_{\sigma\rho} & D_{\sigma\sigma} \end{vmatrix} = \begin{vmatrix} D_{\rho\sigma} & D_{\rho\tau} \\ D_{\sigma\sigma} & D_{\sigma\tau} \end{vmatrix} = \begin{vmatrix} D_{\sigma\sigma} & D_{\sigma\tau} \\ D_{\sigma\tau} & D_{\tau\tau} \end{vmatrix} = 0 \qquad \text{(III, 11)}$$

must be fulfilled for all values of ρ, σ, τ, θ which fulfill the first two equations of (III, 9). We must also consider that in each case (III, 4) must be fulfilled.

Section IV. Applications. The True Reproduction Objective

In the book *Strahlenoptik*, the term "true image" was introduced to denote a special case of a sharp image in which $\mathfrak{z}$ is constant for all points of the object surface. That means that the perpendiculars from image and object points to the axis have a ratio which is constant over the entire object surface.

We shall call a special case of true images *affine* images. In this case, there exist axial points in object and image space such that, if the coordinate origins are placed at these points, the ordinates of object and image point have a constant ratio. This means that

$$\alpha = n'z'/nz = \text{Constant}. \qquad \text{(IV, 1)}$$

The so-called axial magnification is constant. In this case, object and image surface can be transformed into each other by an "affine deformation."

$$n'z' = \alpha nz, \quad n'h' = \beta nh. \tag{IV, 2}$$

Wherefore we have chosen the name "affine image." A special case is that in which

$$\alpha = \beta. \tag{IV, 3}$$

We then can speak of a similar image. In this case, object and image surface are similar to each other.

Of course, each image of the infinite plane can be regarded as a similar image with $\alpha = \beta = 0$. Thus, we need not wonder why we get only a simple expression for the eiconal in the case of a similar and affine image.

What about the use of the two eiconals W and D? In finding the eiconal for a special kind of image formation it is usually easier to find W, but if we have W, going over to D gives us at once the qualities of a special eiconal in regard to the images of all points of space.

We ask now for the condition that a given surface

$$z = \phi(h^2) \tag{IV, 4}$$

has an affine image with the two constant magnifications α and β. In (III, 5) and (III, 6) we can regard β as constant and considering (IV, 1), we find in the same way as we constructed D, that W may be regarded as a function of

$$\kappa - \alpha\mu, \quad ((1+\beta^2)/\beta - 2\lambda - (\mu\beta - \kappa)^2/\beta)^{\frac{1}{2}}, \tag{IV, 5}$$

in which, however, α and β are constants.

If we call these variables χ and σ, we have

$$\begin{aligned} nz &= W_\chi, & n'z' &= \alpha W_\chi, \\ nh &= W_\sigma/\beta^{\frac{1}{2}}, & n'h' &= W_\sigma \beta^{\frac{1}{2}} \end{aligned} \tag{IV, 6}$$

and W is any function of σ and χ that fulfills the differential equation

$$W_\chi = \phi((W_\sigma/\beta^{\frac{1}{2}})^2). \tag{IV, 7}$$

In the special case of a similar image, we have, according to (IV, 5)

$$W = \psi(\kappa - \beta\mu, \lambda). \tag{IV, 8}$$

As a special case, we consider first the image of a plane. Eq. (IV, 7) gives $W_\chi = 0$, or

$$W = f(\sigma) = f((1+\beta^2)/\beta - 2\lambda - (\mu\beta - \kappa)^2/\beta)^{\frac{1}{2}}. \tag{IV, 9}$$

Eq. (IV, 9) gives the eiconal for an ideal reproduction objective which serves to give a sharp, undistorted image of a plane with a manification β.

As a second case, we deal with the image formation of an ellipsoid on an ellipsoid. Given that the ellipsoid

$$(nz/C_1)^2 + (nh/C_2)^2 = 1 \tag{IV, 10}$$

shall be imaged on

$$(n'z'/\alpha C_1)^2 + (n'h'/\beta C_2)^2 = 1, \tag{IV, 11}$$

Eq. (IV, 7) gives

$$W_\chi^2/C_1^2 + W_\sigma^2/\beta C_2^2 = 1 \tag{IV, 12}$$

as differential equation for all eiconals which image our surfaces on each other. Furthermore, we shall investigate the eiconal for a centric-symmetric system.

Boegehold and I found that in this case W can be regarded as a function of λ alone. We now go over to the diapoint eiconal. Here we find, if we put the origin at the center

$$\begin{aligned} D &= f((1+\theta^2)/2\theta - (\tau\theta + \rho)^2/2\theta - \sigma^2/2) \\ &= f(\chi). \end{aligned} \tag{IV, 13}$$

Differentiation gives

$$\begin{aligned} nz &= W_\rho = -(\tau\theta + \rho) f_\chi/\theta, \\ nh &= W_\sigma/\theta^{\frac{1}{2}} = -\sigma f_\chi/\theta^{\frac{1}{2}}, \\ n'z' &= -(\tau\theta + \rho) f_\chi, \\ n'h' &= -\sigma\theta^{\frac{1}{2}} f_\chi, \end{aligned} \tag{IV, 14}$$

from (14) we get

$$n'z' = \theta nz, \quad n'h' = \theta nh, \tag{IV, 15}$$

or, expressed verbally: Object point and diapoint lie on the same straight line through the origin. We now ask, according to Section III, which points have a rotation-symmetric and which points have a sharp image. We find that

$$
\begin{aligned}
D_{\rho\rho} &= ((\tau\theta+\rho)/\theta)^2 f_{\chi\chi} - f_\chi/\theta, \\
D_{\rho\sigma} &= (\tau\theta+\rho)\sigma f_{\chi\chi}/\theta, \\
D_{\rho\tau} &= (\tau\theta+\rho)^2 f_{\chi\chi}/\theta - f_\chi, \\
D_{\sigma\sigma} &= \sigma^2 f_{\chi\chi} - f_\chi, \\
F_{\sigma\tau} &= \sigma(\tau\theta+\rho) f_{\chi\chi}, \\
D_{\tau\tau} &= (\tau\theta+\rho)^2 f_{\chi\chi} - \theta f_\chi.
\end{aligned} \qquad \text{(IV, 16)}
$$

That shows first, according to

$$
\begin{vmatrix} D_{\rho\rho} & D_{\rho\sigma} & D_{\rho\tau} \\ D_{\sigma\rho} & D_{\sigma\sigma} & D_{\sigma\tau} \\ D_{\tau\rho} & D_{\tau\sigma} & D_{\tau\tau} \end{vmatrix} = \begin{vmatrix} D_{\rho\rho} & D_{\rho\sigma} & \theta D_{\rho\rho} \\ D_{\sigma\rho} & D_{\sigma\sigma} & \theta D_{\sigma\rho} \\ D_{\tau\rho} & D_{\tau\sigma} & \theta D_{\sigma\tau} \end{vmatrix} = 0 \qquad \text{(IV, 17)}
$$

that all points have a rotation-symmetrical caustic; moreover, from (IV, 15) that the axis of symmetry is a straight line.

We ask what points have a sharp image. We find the three Eqs. (III, 11) lead to

$$f_\chi f_{\chi\chi}[(1+\theta^2)/\theta - 2\chi] = f_\chi^2$$

for all values of θ and χ for which

$$nz = -(\tau\theta+\rho)f\chi/\theta, \quad nh = -\sigma f_\chi/\theta^{\frac{1}{2}} \qquad \text{(IV, 18)}$$

are constant. Then, we must have

$$n^2z^2 + n^2h^2 = (1/\theta)[(1+\theta^2)/\theta - \chi/2] = C^2 \qquad \text{(IV, 19)}$$

constant. We have a true image of a sphere with the radius C, and a constant magnification

$$\theta = \beta_0. \qquad \text{(IV, 20)}$$

Eq. (IV, 20) shows that we can interchange θ and $1/\theta$. Therefore, we must also have a constant image of a second sphere with the magnification $1/\beta_0$. That will be still more intelligible later on.

We put into (IV, 18) $\theta = \beta_0$ and integrate. We get

$$
\begin{aligned}
D &= C((1+\beta_0^2)/\beta_0 - 2\chi)^{\frac{1}{2}} \\
&= C((1+\beta_0^2)/\beta_0 - (1+\theta^2)/\theta \\
&\qquad + (\tau\theta+\rho)^2/\theta + \sigma^2)^{\frac{1}{2}}.
\end{aligned} \qquad \text{(IV, 21)}
$$

If we now go back to the angle eiconal, we get, according to (IV, 1)

$$W = C((1+\beta_0^2)/\beta_0 - 2\lambda)^{\frac{1}{2}} \qquad \text{(IV, 22)}$$

as the eiconal of a centric system, which gives a sharp image of the two spheres given by

$$
\begin{aligned}
&(nz)^2 + (nh)^2 = \beta_0 C^2 \qquad (n'z')^2 + (n'h')^2 = C^2/\beta_0, \\
&\qquad\qquad \text{or} \qquad\qquad \text{(IV, 23)} \\
&(nz)^2 + (nh)^2 = C^2/\beta_0 \qquad (n'z')^2 + (n'h')^2 = \beta_0 C^2.
\end{aligned}
$$

The eiconal for the single sphere, which must be a special case of our systems, is indicated by

$$\beta = n/n', \quad C = r(nn')^{\frac{1}{2}}$$

or

$$W = r(n^2 + n'^2 - 2nn'\lambda)^{\frac{1}{2}}. \qquad \text{(IV, 24)}$$

The second sphere with the sharp image is here the aplanatic sphere.

An Interesting Optical Law

M. HERZBERGER, *Eastman Kodak Research Laboratories, Rochester, New York*

(Received February 25, 1936)

LET us consider a rotation symmetric optical system. We take a skew ray, a (Fig. 1), and its image ray a'. If they do not intersect each other, we can project the image ray, a', parallel to the axis until its projection, b, meets the object ray at O. The plane E, through a, b might intersect the axis at a point C, which we call the axial center point. The line CO then may form, with object and image ray, angles ϵ and ϵ'.

We can now demonstrate that, for any ray

$$n \sin \epsilon = n' \sin \epsilon', \tag{1}$$

where n, and n' are the refractive indices of object and image space, respectively.

In the case of a single surface, this law is identical with the Snellius-Descartes law of refraction. But it can easily be demonstrated that it holds true for an entire system if it holds true for the component parts.

A generalization of this theorem can be stated as follows:

Instead of shifting the image ray, a', we can shift the object ray, a, parallel to the axis, until its projection, b', intersects the image ray. Thus, we get the plane E', the image center point C' in which this plane meets the axis, and the point O' in which image ray and projection intersect. Of course, OC and $O'C'$ are parallel.

We now consider an arbitrary meridian plane (a plane through the axis). It can intersect the object ray in B, the image ray in B'. Then, CB and $C'B'$ are parallel to each other. We have then for the angles η, η', which CB forms with object and image ray:

$$n'\beta' \sin \eta' = n \sin \eta, \tag{2}$$

where $\beta' = C'B'/CB$.

The connection between B and B' has an important optical significance. We have called B' the diapoint of B (see preceding paper "A New Theory of Optical Image Formation") and *vice versa*, and have shown that the investigation of the diapoints of an object point gives all necessary information about the image caustic. The magnitude β' defined above is called diamagnification. It is equal to the ratios of the perpendiculars from point and diapoint on the axis.

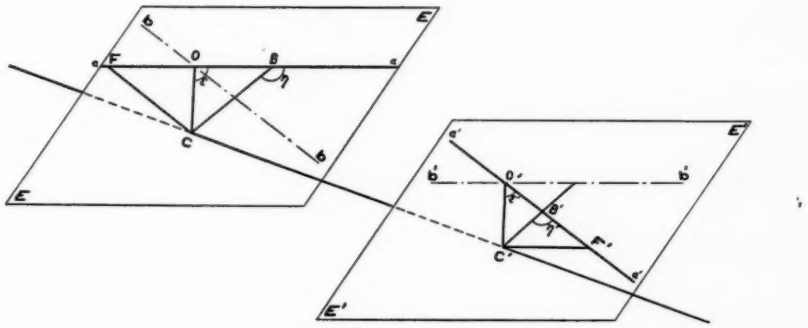

FIG. 1

The Optics of Nerve Myelin

RICHARD S. BEAR AND FRANCIS O. SCHMITT, *Department of Zoology, Washington University, St. Louis*

(Received February 22, 1936)

Qualitative studies on nerve birefringence have in the past indicated many important facts regarding the ultrastructure of the individual axons. In order to apply the sensitive optical method in greater detail to such investigations quantitative methods are necessary. It is also desirable that such procedures be applicable to the axons under physiological conditions. The myelin sheaths of the nerve fibers are most readily studied optically in this way because of their high birefringence, but the chief difficulties in the interpretation of the observations result from the radial orientation of the optic axes of the sheath elements. In this paper are presented simple mathematical considerations leading to an expression relating the experimentally measured sheath retardation and axon and sheath dimensions to the birefringence of the sheath, calculated as an intrinsic property of the sheath elements. Consideration of possible theoretical errors leads to confidence that the results obtained are accurate within a few percent. Typical frog sciatic axons show a birefringence of about 0.011, which is a fairly large value for biological materials. The results supplement x-ray evidence for a high degree of molecular or micellar orientation in the myelin sheath.

THE qualitative description of the optical characteristics of the nerve myelin sheath has long been known. The researches of Valentin,[1] Klebs,[2] and v. Ebner[3] have demonstrated that the sheath elements of medullated (e.g., most frog and mammalian) axons behave as would small uniaxial crystallites with optic axes oriented radially about the nearly isotropic central axis cylinder. Since these early investigations practically no attempts have been made to apply the valuable optical method to the study of nerve ultrastructure in a quantitative way.

The extremely fine and fragile axons can be studied under the polarizing microscope only from the lateral direction if one requires that they be in as nearly normal condition as possible. This fact is perhaps responsible for the lack of quantitative data mentioned above, for, in attempting under these conditions to obtain a figure for the birefringence of sheath elements as an intrinsic property, it is found that of the two experimental quantities necessary, i.e., relative retardation of the two light components (extraordinary and ordinary rays) and the appropriate length of optical path, only the former can be measured directly. The "path length" to be employed is no obvious function of axis cylinder and sheath dimensions, because of the radial orientation of the birefringent elements, and the proper function to be used must be determined from theoretical considerations. This problem is considered herein from the standpoint of the simple optical principles involved. Discussion of experimental results from the point of view of their bearing on the structure and physiology of nerve is reserved for other more appropriate places.

RETARDATION IN A SINGLE SHEATH ELEMENT

For purposes of orientation the optical characteristics of the various components of a typical medullated axon are diagrammed in Fig. 1. The inner axis cylinder, *A*, is surrounded by the myelin sheath, *B* (the sheath thickness is drawn larger than is actually the case). *a* and *c* are cross sections of refractive index ellipsoids for the sheath elements as viewed from the side at axon edges, *b* the same as seen in the center, in which case observations are made looking down the optic axes of the sheath elements. The lines in the sheath cross section indicate orientations of the optic axes of the positive uniaxial sheath elements. The axis cylinder is also optically anisotropic, being uniaxial and positive with respect to the longitudinal direction of the axon. *d* is the cross section of the index ellipsoid of the axis cylinder as it appears in a lateral view, while *e* represents the circular section indicative of the isotropic longitudinal appearance. The shapes of the ellipses do not indicate quantitative relationships; the anisotropy of the axis cylinder is

[1] G. Valentin, *Die Untersuchung der Pflanzen und der Thiergewebe* (Leipzig, 1861), pp. 293–303.
[2] E. Klebs, Virchow's Archiv **32**, 168 (1864).
[3] V. v. Ebner, *Untersuchungen über die Ursachen der Anisotropie organisierter Substanzen* (Leipzig, 1882), pp. 180–191, 232.

much less than that of the myelin sheath. As viewed laterally in the diagonal position between crossed Nicols the sheath appears very bright at the axon edges and only faintly illuminated in the region of the axis cylinder, for reasons which will be evident from the diagram. Longitudinal observations show the axis cylinder isotropic, while in the sheath region appears a brilliant "positive cross." Under the usual conditions for the examination of birefringent materials with the polarizing microscope one illuminates the object by means of a number of plane polarized rays, each of which may be regarded as being split into "extraordinary" and "ordinary" plane polarized components under the influence of the optical anisotropy of the substance. These components, in the case of a nerve axon, during passage through the sheath meet the optic axes of the myelin elements at angles, θ, which vary from point to point. The velocity of the ordinary ray is, of course, constant and independent of θ, but the extraordinary ray's velocity depends on θ in a way which may be determined as follows.

We consider a representative element of the myelin sheath and construct a velocity ellipsoid about any point of the extraordinary ray's path in the element, after the manner of the Huygens treatment of uniaxial crystals. The velocity of propagation of the extraordinary ray in a direction paralleling the optic axis of the element is c/n_0, while in directions perpendicular thereto it is c/n_a, c being the velocity of light and n_a and n_0 the extraordinary and ordinary indices of refraction. With these values for the principal axes of the velocity ellipsoid it is readily apparent that

$$v_e^2(n_a^2 \sin^2 \theta + n_0 \cos^2 \theta) = c^2 \qquad (1)$$

is the equation for the velocity ellipsoid relating the velocity of the extraordinary ray v_e to the angle θ which its direction of propagation makes with the optic axis.

If we let

$$n_0/n_a = (1-\rho)^{\frac{1}{2}}, \qquad (2)$$

where ρ is a small number expressing the departure of the ratio n_0^2/n_a^2 from unity, then from (1)

$$n_e = c/v_e = n_a(1-\rho \cos^2 \theta)^{\frac{1}{2}}, \qquad (3)$$

in which n_e is the "effective" index of refraction of the myelin element with respect to the extraordinary ray.

Assuming that an associated pair of extraordinary and ordinary rays are traveling through the sheath along the same path, we can then write for the small retardation of the extraordinary ray with respect to the ordinary ray resulting from passage through the element:

$$d\Gamma = (n_e - n_0)ds = [n_a(1-\rho \cos^2 \theta)^{\frac{1}{2}} - n_0]ds, \qquad (4)$$

ds being the path length in the element.

A convenient simplification of (4) can be accomplished by expanding the quantities under the radicals in (2) and (4). To the approximation that powers of ρ higher than the first are negligible compared to unity and by elimination of ρ from the equations thus obtained, it is found that:

$$d\Gamma = (n_a - n_0) \sin^2 \theta ds. \qquad (5)$$

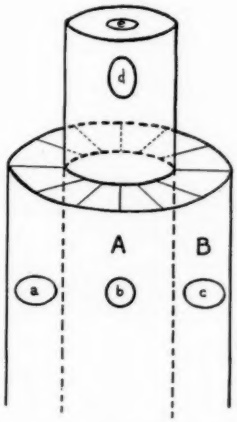

FIG. 1. Diagram of a medullated nerve axon from a lateral viewpoint.

INTEGRATION OF THE EFFECTS OF MANY ELEMENTS

Before attempting to integrate the retardations of elements in order to obtain an expression relating the observed total retardation to the quantity $n_a - n_0$, which measures the birefringence of the sheath material, it is necessary to consider briefly the method by which the microscopic observations are carried out. These

are performed under "orthoscopic" conditions, i.e., any point of the field of view as it is impressed upon the retina of the observer appears as the resultant of all rays which are able to pass through the actual corresponding point in the object and into the microscope objective. Rays that pass through a given object point travel in various directions whose maximum departure from parallelism with the microscope axis is limited chiefly by the working aperture of the system. This difficulty, combined with the complications introduced by reflection and refraction effects at the sheath surfaces, would be serious were it not for the fact, as will be shown later, that for rays which appear to come from the sheath (the bright edges of the axon in the lateral view) the direction of transmission is not too critical in determining the total retardation.

If we assume this to be true for the moment, it becomes apparent that the simplest procedure in integrating (5) is to consider paths through the axon which are perpendicular to the focal plane. First we imagine a cylinder composed entirely of material having the same composition and radial orientation of optic axes as actually exists in the nerve axon sheath. The problem is illustrated by Fig. 2, which is a cross section of the cylinder whose axis is perpendicular to the plane of the paper at O. The diameter through O represents the plane upon which the microscope is focused; P is a point under observation, located a distance r_0 from O, the radius of the cylinder being r_1. At P is erected a perpendicular to OP which represents the ray path through the cylinder and intersects the cylinder surface at the points r_1, Φ_1 and r_1, Φ_2. Polar coordinates r and Φ are used to express the linear path, where Φ is an angle referred to the line OP, the equation of the path being

$$r_0/r=\cos\Phi. \tag{6}$$

The element of path length is obviously:

$$ds=(r^2+p^2)^{\frac{1}{2}}d\Phi, \tag{7}$$

while from the previous definition of θ,

$$\tan\theta=rd\Phi/dr=r/p,$$

or

$$\sin^2\theta=r^2/(p^2+r^2), \tag{8}$$

p in all cases representing $dr/d\Phi$. Combination

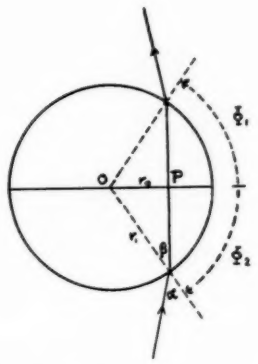

FIG. 2. Cross section of cylinder composed of axon sheath material. For explanation see text.

of Eqs. (5), (6), (7), and (8) readily yields:

$$d\Gamma=(n_a-n_0)r_0d\Phi. \tag{9}$$

Integration of (9) is simple and leads to

$$\Gamma=(n_a-n_0)r_0(\Phi_1-\Phi_2), \tag{10}$$

where Γ is the total retardation of the extraordinary ray with respect to the associated ordinary ray produced by the complete path from r_1, Φ_2 to r_1, Φ_1. Since in this case $\Phi_2=-\Phi_1$ and $\cos\Phi_1=r_0/r_1$, Eq. (10) may be written

$$\Gamma/(n_a-n_0)r_1=2f_1\cos^{-1}f_1, \tag{11}$$

in which $f_1=r_0/r_1$. In this form the right-hand term includes factors which may be regarded as being independent of the cylinder size.

If now we introduce a practically isotropic axis cylinder of radius r_2 ($<r_1$) without allowing for the refraction at the sheath-axis cylinder boundary, it is readily observed that the effect will be that of subtracting from the retardation as given by (11) an amount equal to that which would have been produced by a cylinder of sheath material of radius r_2. The equation similar to (11) for the case of combined axis cylinder and sheath under these conditions is:

$$\Gamma/(n_a-n_0)r_1=2f_1(\cos^{-1}f_1-\cos^{-1}(f_1/g)), \tag{12}$$

where $g=r_2/r_1$.

DISCUSSION

The form of Eq. (12) is shown in Fig. 3. Its interesting feature is the maximum, which corre-

sponds to the experimentally observed maximum in the birefringence occurring at the axon sheath. The theoretical maximum, however, occurs at the axis-cylinder-sheath boundary, and is therefore not strictly comparable to experiment, which indicates a maximum within the sheath. For this reason, in actual calculations a modification of (12)—Eq. (18)—is employed. Preliminary results with this method have indicated certain facts of importance to the structure of nerve, and it is therefore necessary to consider in detail the assumptions upon which Eq. (12) is based in order that we may be certain that the results calculated from the theoretical expression are not too greatly influenced thereby.

The treatment, to the present point, has depended on two principal assumptions: (1) that the associated extraordinary and ordinary rays follow the same path, and (2) that the observations for points in the focal plane are essentially those to be obtained from transmission of parallel rays through the axon perpendicular to the focal plane without allowing for refraction and reflection effects. Complete determination of the adequacy of these suppositions can hardly be found without knowledge beforehand of the optical characteristics of the materials involved. The probable degree of importance of these assumptions can be shown, however.

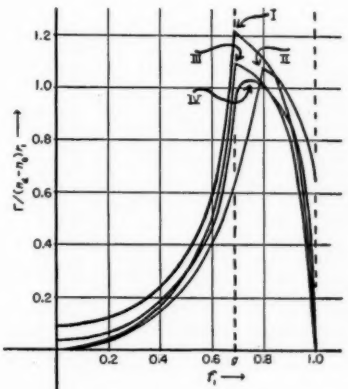

FIG. 3. Theoretical curves for $\Gamma/(n_a-n_0)r_1$ plotted against f_1. The different curves correspond to values obtained for various angles made by the ray paths with the focal plane. The maxima are marked thus: I, $\Omega=70°$, Eq. (16); II, $\psi=70°$, Eq. (15); III, $\psi=\Omega=90°$, Eqs. (12), (15) or (16); and IV, average of ordinates for the other three curves. The dotted lines indicate the sheath boundaries. All curves are for $g=0.69$.

It is frequently assumed that for thin layers of birefringent materials the separation of the extraordinary and ordinary components is negligible. This would, therefore, seem justifiable in the present case, for the largest possible path length is not greater than the diameter of the axon, which is usually 20μ or less in medullated nerve axons. Here, however, there are two factors tending to cause departure of the extraordinary from its associated ordinary ray: (1) the different angles at which the two are refracted upon entering the sheath, and (2) the continual curving of the extraordinary ray as it meets the sheath elements in different directions with respect to their optic axes. We shall consider the case of transmission of the two components in a cross-sectional plane, since then the rate of change of the angle θ with path length, hence possible curvature of the extraordinary ray, is greatest.

We again imagine a cylinder entirely composed of sheath material having the same description as given before (Fig. 2), the point of incidence from the exterior being at r_1, Φ_2 and the path of the extraordinary ray $r=f(\Phi)$. Since $dt=ds/v_e$ is the time required for the ray to travel the element of path ds, it follows from Eqs. (3), (7), (8) and integration that the time required for the extraordinary ray to pass from the point of incidence to the point r, Φ is

$$t=\frac{n_a}{c}\int_{\Phi_2}^{\Phi}[p^2(1-\rho)+r^2]^{\frac{1}{2}}d\Phi.$$

Fermat's law indicates that for the real path this time is an extremum, or $\delta t=0$. Applying the calculus of variations one finds as differential equation for the path:

$$rdp/d\Phi-2p^2=r^2/(1-\rho),$$

whose solution is

$$1/r=c_1\cos[\Phi/(1-\rho)^{\frac{1}{2}}]+c_2\sin[\Phi/(1-\rho)^{\frac{1}{2}}].$$

The constants of integration, c_1 and c_2, can be determined from the boundary conditions: (1) that r_1, Φ_2 is a point on the path, and (2) that the angle of refraction β (equal to θ at r_1, Φ_2) and the angle of incidence α at the point of incidence, r_1, Φ_2, are related by

$$\sin\beta=\frac{\sin\alpha}{(n_0^2/n_1^2+\rho\sin^2\alpha)^{\frac{1}{2}}},$$

where n_1 is the refractive index of the medium outside the cylinder. (The refraction equation is that for the extraordinary ray at a plane surface of a uniaxial material whose optic axis is perpendicular to the surface. Note that when $\rho=0$ it reduces to Snell's law, which the ordinary ray obeys.) The final equation for the path of the extraordinary ray is:

$$\frac{1}{r_e}=\frac{1}{r_1\sin\alpha}\left[\sin\alpha\cos\frac{(\Phi_2-\Phi)}{(1-\rho)^{\frac{1}{2}}}+(\cos^2\alpha+\nu-\rho\cos\alpha)^{\frac{1}{2}}\sin\frac{(\Phi_2-\Phi)}{(1-\rho)^{\frac{1}{2}}}\right], \tag{13}$$

where the subscript e distinguishes the radius coordinate of the extraordinary ray and $\nu=(n_0/n_1)^2-1$. When $\rho=0$ we have for the path of the ordinary ray:

$$\frac{1}{r_0}=\frac{1}{r_1\sin\alpha}[\sin\alpha\cos(\Phi_2-\Phi)+(\cos^2\alpha+\nu)^{\frac{1}{2}}\sin(\Phi_2-\Phi)], \tag{14}$$

which is the equation of a straight line. It is apparent that (13) and (14) are practically identical for small ρ. Since the values of ρ corresponding to reasonable birefringence of the axon sheath are small (e.g., $\rho\leqq0.02$ for several experimental cases to be cited later) it follows that errors introduced by assuming identity in the paths of the two components will be negligible.

In considering rays which pass through the axon in directions not perpendicular to the focal plane we investigate two extreme cases of transmission through the point P. We consider (1) all rays which lie in a cross-sectional plane through P, and (2) the rays in a plane parallel to the axon axis and perpendicular to the cross-sectional plane at P. The intersection of these two planes is the perpendicular to the focal plane at P previously considered. Mathematical treatment of these two cases is carried out in a manner similar to that employed for the perpendicular and resulting in Eq. (12). The corresponding results are:

$$\frac{\Gamma}{(n_a-n_0)r_1}=f_1\sin\psi\left[\cos^{-1}(2f_1^2\sin^2\psi-1)-\cos^{-1}\left(\frac{2f_1^2}{g^2}\sin^2\psi-1\right)\right], \tag{15}$$

$$\frac{\Gamma}{(n_a-n_0)r_1}=2\left\{\left(\frac{1}{\sin\Omega}-\sin\Omega\right)\left[(1-f_1^2)^{\frac{1}{2}}-\left(1-\frac{f_1^2}{g^2}\right)^{\frac{1}{2}}\right]+f_1\sin\Omega\left(\cos^{-1}f_1-\cos^{-1}\frac{f_1}{g}\right)\right\}. \tag{16}$$

Here the angles ψ and Ω express the angle between the path and the focal plane in the cross-sectional and longitudinal planes, respectively. For ψ and Ω equal to 90°, Eqs. (15) and (16) are equivalent to Eq. (12).

The only experimental quantity involved in the right-hand sides of (15) and (16) is g, the ratio of the axis cylinder and axon radii. This, in a typical example, was measured to be 0.69. Using this value for g, one can plot curves for $\Gamma/(n_a-n_0)r_1$ against f_1, which ranges from 0 to 1. This has been done in Fig. 3 for values of 70° and 90° for ψ and Ω (the 90° curves coinciding, of course). From the facts that the sine functions of ψ and Ω are the only ones occurring in (15) and (16) it is apparent that letting ψ or Ω equal 110° will give rise to no new curve. This is a result of the symmetry of the axon above and below the focal plane.

In order to see qualitatively the effect of the rays going through P the ordinates of the three curves of Fig. 3 have been averaged to obtain the curve without the discontinuous change in slope at the axis cylinder-sheath boundary. It is interesting to note (1) that the maxima in all curves occur within the "pure sheath" region (i.e., at or beyond $f_1=g$), (2) that the maxima are all approximately the same height, and (3) that the average curve is approximately that for ψ or Ω equal to 90°, i.e., the curve following Eq. (12) previously developed. The chief differences between the average curve and that for

perpendicular transmission are a rounding off and a slight lowering of the maximum. In regarding Fig. 3 it is important to bear in mind that in practice it is the retardation corresponding to the maximum which is measured.

The curves of Fig. 3 are calculated from considerations which neglected refraction and reflection at the sheath surfaces. These effects during entry of the rays into the sheath can only have the result of shifting the average curve to a position more nearly resembling a curve for ψ less than 90°, and it is evident that even if this occurs to the extent that the average path becomes similar to that for $\psi = 70°$, the height of the maximum is not greatly changed. Reflection and refraction at the inner sheath surface may remove the maximum farther into the "pure sheath" region. Experimentally it is observed that the maximum is located at a position which is approximately one-third the sheath thickness for the axis cylinder-sheath boundary, i.e., for

$$f_1 = g + (1-g)/3 = (2g+1)/3. \qquad (17)$$

It can be seen that combination of (17) with (12) will give a value for $\Gamma_{max}/(n_a - n_0)r_1$ not far from that of the maximum of the "average" curve. Accordingly we employ for the calculations of $n_a - n_0$:

$$n_a - n_0 = \frac{3\Gamma_{max}}{(d_1+2d_2)\cos^{-1}((d_1+2d_2)/3d_1)}, \qquad (18)$$

which results from combination of (12), or (11), and (17), when diameters d_1 and d_2, of axon and axis cylinder, respectively, are used instead of radii.

The assumptions involved in combining (17) and (12) for the maximum will probably not involve an error in the calculated value of $\Gamma_{max}/(n_a - n_0)r_1$ of more than a few percent. Since the applications of principal interest are comparisons between axons under the same conditions, this error is even less important if the same relation is used in all cases.

Eq. (18) automatically takes account of any variations in g from axon to axon. Curves similar to those in Fig. 3 have been calculated for values of g other than 0.69, and it can be shown that approximately the same considerations apply as discussed above. In addition, relatively unimportant variations in g have been observed experimentally.

Experimental Considerations

Reflection and refraction effects cannot very well be treated with greater exactness than is employed above without knowledge of the mean refractive indices of the axis cylinder and myelin sheath. It is difficult to see how these can be determined experimentally, since immersion methods would have to be employed and the delicate colloidal state of the axon components would be greatly upset by immersion in fluids other than Ringer solution or serum. One can, however, obtain qualitative information from ordinary microscopic examination of the axons.

The axis cylinder is a weak solution of proteins and salts, hence does not have an index much greater than that of the Ringer solution. The sheath index is somewhat higher than that of either of its neighboring fluids, as can be determined from the facts that when the microscope is focused above the median plane of the axon the sheath is brighter than its contiguous materials; when focused below, the sheath is darker. That the sheath is not of excessively high refractive index would seem to follow from the fact that from the lateral view the axon sheath is observed to pass light up to fairly large angles of incidence.

A qualitative examination with regard to the magnitude of birefringence at various points from the axon center to the edge shows that the "average" curve of Fig. 3 is a fair representation of the effects observed, except that in the immediate neighborhood of points for which $f_1 = g$ or 1 (i.e., at sheath boundaries) refraction and reflection disturb the situation, lowering the apparent retardation considerably. That this does not extend very far into the sheath is shown by the fact that the maximum for birefringence occurs very near to the position indicated by the curve. This fact has led to the choice of f_1 as expressed by Eq. (17).

It is also of importance to note that working aperture and conditions of illumination apparently do not influence the value obtained experimentally for Γ_{max}, indicating that the difficulties imposed by the "orthoscopic" method of examination and the reflection and refraction effects

are not too significant. Under improper illumination, diffraction effects resulting from the small thickness of the sheath (3μ or less) become very marked, but the previous considerations still apply to the principal diffraction intensity maximum.

The method of measurement of d_1 and d_2 consists in the employment of a *camera lucida* with total magnification of about 1350 times, as determined from calibration with a stage micrometer. Instruments useful for the determination of Γ_{max} are the $\frac{1}{4}\lambda$ plate (in conjunction with rotation of the analyzer, after the method of Sénarmont) and, for retardations of less than $(1/20)\lambda$, Köhler's rotating mica plate of small retardation. White light may be employed successfully in both cases, as has been shown by comparison with results obtained by illumination with a sodium vapor lamp.

Axons are teased from whole nerves and examined at the same region for fiber and sheath dimensions and maximum total retardation. Eq. (18) is then used to calculate $n_a - n_0$ for the myelin sheath.

Since experimental results are to be treated in another place in greater detail, it is sufficient to indicate here that the average birefringence for sheaths of a number of fibers from sciatic nerves of various frogs is 0.011 for the size range 9 to 17μ (diameters).[4] This is an order of magnitude higher than the birefringence of similar nervous tissue measured as that of a whole bundle of fibers, and shows that the myelin sheath is quite well organized, as x-ray diffraction evidence also indicates.[5]

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[4] ρ of Eq. (2) is approximately twice the birefringence, hence $\rho \leqq 0.02$. The neglect of terms containing ρ^2 and higher powers in deriving Eq. (5) is therefore justified.

[5] F. O. Schmitt, R. S. Bear and G. L. Clark, Radiology **25**, 131 (1935).

The Vacuum Cell Luminescence Microscope and Its Use in the Study of Luminescent Materials

JOHN GALLUP, *RCA Radiotron Division, RCA Manufacturing Co., Inc., Harrison, New Jersey*

(Received January 18, 1936)

A new type of luminescence microscope is described. Its use in the examination and comparison of materials luminescing under electron bombardment in vacuum is explained. Fields of self-luminous particles are obtained of sufficient brightness to permit of satisfactory visual observation at magnifications up to 500×. Good photomicrographs can be made at 100× with ordinary film. In the study of manganese-activated calcium silicates, this microscope has facilitated the determination of many of the relationships which exist between color of luminescence, intensity of luminescence and crystal form. The low temperature form of calcium metasilicate luminesces more brightly than any other calcium silicate studied. A thorough wet mixing of raw materials is necessary to obtain a product of uniform luminescence.

INTRODUCTION

DURING recent work on the microscopic identification of luminescent calcium silicates (manganese activated) intended for use in the luminescent screens of cathode-ray tubes, it became apparent that much time could be saved and much more information obtained if the materials could be examined at high magnifications illuminated by their own emitted light. Such observation should reveal external crystal forms and tie up crystal structure with colors and relative intensities of luminescence. In order to obtain such a self-illuminated field it would be necessary to provide some means of exciting the particles of the material to luminescence on the microscope stage. Since x-rays, ultraviolet light, and cathode rays furnish the principal means of exciting materials to luminescence, one of these would need to be employed. However, since equipment for x-ray bombardment or for ultraviolet illumination is quite expensive and somewhat complicated, a self-illuminated field at any considerable magnification could best be attained cheaply and easily with an ordinary microscope by viewing the materials in vacuum under bombardment from a high frequency discharge. This method would have the further advantage of showing the material under approximately the same form of excitation as that under which it is used in cathode-ray tubes. This is important since a material may exhibit different colors and intensities of luminescence under different forms of excitation. With these considerations in mind, the author devised the cell described here.

THE VACUUM CELL

Essentially, the luminescence cell consists of a small cup-shaped bowl with a removable cover glass and two electrodes between which the sample is placed. Provision is made to evacuate the cell with a pump to a pressure of the order of 1 mm of mercury. Bombardment of the sample by cathode rays is produced by the high frequency discharge of a spark coil. The cell can be made with the exhaust tube arranged to pass down through the microscope stage, or with this tube sealed at the side of the cell so that the assembly can be supported by a clamp rather than by the microscope stage as it is in the first case. In either event, the cell can easily be made in a short time by an experienced glass blower.

Fig. 1 shows the details of a satisfactory construction. The vacuum space A is about 5/8 of an inch in diameter and 1/4 of an inch in depth. The lower electrode E is a coiled nickel wire on which rests the metal cup containing the sample. This wire is welded at J to a tungsten wire I sealed in the glass tubing below the microscope stage. The tungsten wire goes directly to the spark coil. The cell should be so constructed that no part of the lower electrode or its connection to the spark coil approaches closer than an inch to the metal of the grounded microscope stage. The glass tube F which leads to the vacuum pump is made from capillary tubing in order to

increase the resistance to ground through the vacuum pump, and thus keep the loss from this source at a minimum. The cover glass *C* is made extra large and heavy so that it does not overheat from the spark discharge and soften the grease in the vacuum seal. Such softening would cause the spark discharge to puncture the seal and to ground, around the edge of the cover glass, to the upper electrode *D*. A good abrasive for grinding the cover glass to fit the cell wall is obtained by placing 600-mesh silicon carbide in heavy stopcock grease. The upper electrode *D* is formed so that only the central ring is in contact with the top of the cover glass. This removal of the remaining portions of the spring from the proximity of the vacuum space avoids spreading the field set up between the cup and ring electrodes. The glass supporting arms *H* are glass rods sealed to the cell and flattened at their outer ends to facilitate clamping them to the microscope stage.

Operation

When a sample is being prepared for viewing, a small amount of the material is spread in a shallow metal cup and placed in the cell on the lower metal electrode. The cover glass is placed on the previously greased edge of the cell and the vacuum pump started. The ground contact is then clipped to the upper electrode and the spark coil turned on as soon as the pump works with a somewhat sharp or metallic sound. The luminescent material will glow strongly. The pump must be turned off at this stage, or the vacuum will become too high for bombardment of the luminescent material to occur at the voltage obtained from the spark coil. If this happens, the admission of a small amount of air through a stopcock in the vacuum line will start the luminescence again. If the ground-glass joint has been properly greased, the luminescence will continue unchanged at a high level of efficiency for four or five minutes without attention. The microscope is focused in the usual manner. The luminous particles stand out sharply against the dark background of the metal cup. This enables crystal forms to be seen clearly. The comparative colors and intensities of different luminescent materials may readily be determined by placing samples of each in the same field. Colors of particles below 0.5μ across can be seen.

Magnifications up to 250× are obtained with a 16-mm objective (10×) used with oculars up to 25× magnification. High magnifications require a thinner cover glass than the one shown in the drawing (2.5 mm). A 9-mm objective (working distance 2 mm) is used with a cover glass 1 mm thick to produce satisfactory fields for visual observation at magnifications up to 500×. Good photomicrographs can be made at magnifications up to 100× with a 2-minute exposure on Eastman D-C ortho plates (see Fig. 2). This exposure time can be considerably shortened by using panchromatic film.

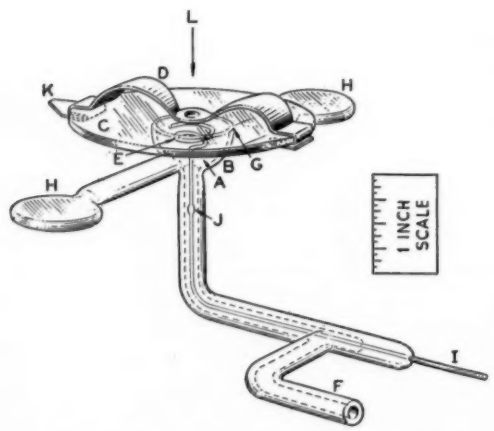

FIG. 1. Vacuum cell. *A*, vacuum space; *B*, glass cup; *C*, cover glass; *D*, upper electrode of spring steel; *E*, lower electrode of coiled nickel wire; *F*, capillary tubing to vacuum pump; *G*, ground-glass joint sealed by grease; *H*, glass arms flattened at ends for clamping to microscope stage; *I*, tungsten wire to coil; *J*, weld; *K*, contact to ground; *L*, direction of viewing.

THE LUMINESCENCE OF CALCIUM SILICATES

A considerable number of samples of calcium metasilicates[1] have been examined, but so far, only a few activated orthosilicates[2] have been studied. The crystal forms[3] were identified by petrographic examination. Many of the first batches of the metasilicates were found to contain much dead material mixed with long needles

[1] Made by H. W. Kaufmann, RCA Radiotron Division, RCA Manufacturing Co., Inc.

[2] Made by H. W. Leverenz, RCA Victor Division, Camden, New Jersey.

[3] A. N. Winchell, *Microscopic Characters of Artificial Minerals* (John Wiley & Sons, New York, 1931).

which luminesced a bright yellow. After the raw materials were wet-milled together for several hours, all of the fired product luminesced. However, the brightest luminescence in every case was given by the long crystals (Fig. 2). Petrographic determinations disclosed that these were the characteristic lath-shaped crystals of the β or low-temperature form of calcium metasilicate.

All the orthosilicates examined contained large amounts of finely crystalline material, but no batch was composed exclusively of any one crystal form, nor was any free from inert material. Two characteristic colors of luminescence were obtained, a green and a red. A correlation of the results of the luminescence microscope with those from petrographic determinations indicates that the low-temperature form luminesces green while the high temperature forms luminesce red. The red and green are of about equal intensity. Both are far weaker than the yellow of the metasilicate.

FIG. 2. Luminescent flat needles and bundles of needles of calcium metasilicate. Magnification 100×.

Acknowledgment and thanks are due Dr. L. B. Headrick of this laboratory for much valuable help in the development of the vacuum cell to its present form.

Sextant with Improved Filters

E. O. HULBURT, *Naval Research Laboratory*

(Received February 24, 1936)

Polarizing plates, made of polarizing film cemented between glass, provide a new and convenient filter for the navigator's sextant. The celestial filter consists of two polarizing plates, one or both of which may be interposed into the optical path, and when both are in position, one may be rotated with respect to the other to yield the desired reduction in light intensity of the celestial object. The combination is a rapidly adjustable variable filter. The horizon filter likewise consists of two polarizing plates. These fulfill a double function. One plate is set with axis of polarization vertical; this usually sharpens the sea sky horizon and diminishes the glare of the sun path. The addition of the second plate, which is rotatable, provides a rapidly adjustable variable filter.

STANDARD types of navigators' sextants are equipped with two sets of glass color filters. One set comprises the "horizon" filters, which may be interposed between the eye of the observer and the horizon, and the other set the "celestial" filters, which may be placed between the eye of the observer and the celestial object, as sun, star, etc., under observation. Each set of filters usually consists of three or four gray or colored glass filters any one or more of which can be put into position. In using the sextant, one chooses various combinations of horizon filters and celestial filters until best distinctness of the horizon and celestial object is secured. To select the correct combination of filters requires time, which may be troublesome, as, for example, when the sun in partially obscured by moving clouds. In such a case, because of the varying brightness of the sun, a celestial filter combination correct for one instant may be too dense in a succeeding instant. Further, it occasionally happens that none of the possible combinations of filters gives exactly the opacity desired, being either too dense or too transparent.

An improved filter has been devised, the optical transmission of which may be varied quickly and continuously to less than 0.5 percent. It consists of two polarizing plates arranged so that one may be rotated with respect to the other. The filter is shown in Fig. 1; the two polarizing plates are mounted on separate arms in order that either or both may be swung into position in the optical path of the sextant. Plate A, Fig. 1, is fixed in its mount, Plate B can be rotated in its plane by turning the knurled edge of its mount with the fingers.

Polarizing plates were made by mounting polarizing sheet with balsam cement between

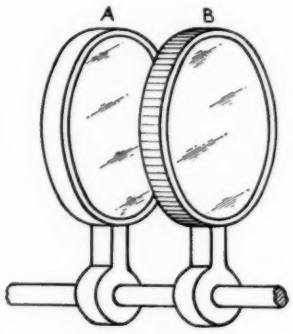

FIG. 1. Polarizing plate filter.

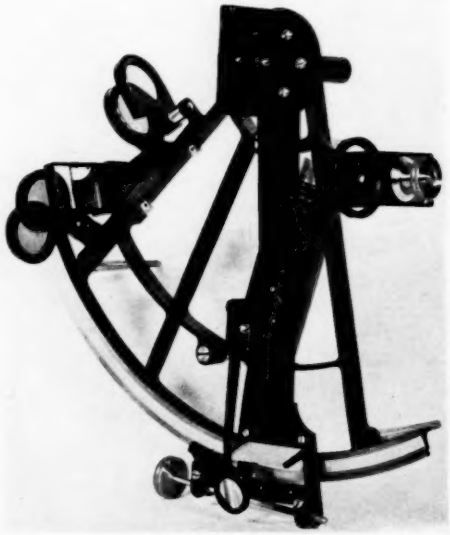

FIG. 2. Sextant with polarizing plate filters.

A A′

B B′

FIG. 3. A, B, unpolarized and A′, B′, polarized photographs of Atlantic Ocean. A, A′, breeze 14 knots; B, B′, breeze 3 knots.

plates of glass. The polarizing plates were a gray color. A single plate transmitted about 30 percent of unpolarized light, two plates together with axes parallel transmitted about 20 percent, and with axes perpendicular less than 0.5 percent. Therefore, with the two plates mounted as in Fig. 1, the following adjustments of light transmission were possible:

	Light Transmission
With no plate	100 percent
With Plate A	30 percent
With Plates A and B, axes parallel	20 percent
Rotating Plate B	Continuously variable from 20 to less than
Axes of A and B perpendicular	0.5 percent

A navigator's sextant equipped with the filter of Fig. 1 in the place of both the celestial filters and the horizon filters is shown in Fig. 2. Light from celestial objects as the sun, moon and stars, is practically unpolarized. Therefore, it makes no difference in which direction the axis of polarization of Plate A of the celestial filter is fixed. When the celestial filter is desired one first puts in A, then B, and finally rotates B until the most satisfactory transmission is obtained.

In the case of the horizon filter, Plate A is fixed in its mount so that when swung into the optical path, its axis of polarization is vertical. As has been previously described,[1] this usually reduces the brilliancy of the sun path on the water, darkens the sea with respect to the sky and renders the horizon more distinct because of the differential amount of polarization of the light from the sea and sky near the horizon. Photographs with and without a polarizing plate over the camera lens are shown in Fig. 3 of the sea toward the sun in breezy and calm weather. In each case, the altitude of the sun was 62°, an altitude at which the polarization effect is nearly at its best. Plate B of the horizon filter may be added to Plate A and rotated to give the desired transmission. In practice, Plate B is seldom necessary.

[1] Hulburt, J. Opt. Soc. Am. **24**, 35 (1934).

Photographic Penetration of Haze

NORA M. MOHLER, *Smith College*

(Received February 7, 1936)

Photographs taken with panchromatic and with infrared plates yield visibility ratios which range from 1.1 for days with thick haze or light rain, to 1.7 for clear days (visibility 20 miles). These values indicate an upper limit of 0.3μ for the radius of drops in the thin haze.

THE penetrating power of infrared plates in comparison with panchromatic plates was used by E. O. Hulburt in a study of the attenuation of light in clear and foggy atmospheres.[1] His results are given for visibilities up to eight miles. This report extends the results to an eye visibility of about twenty miles.

The photographs were taken in the Connecticut valley, from the Mt. Tom range, looking north in most cases. Estimates of visibility were made visually and from the plates by noting the most distant distinguishable object, and checking the distance on a topographic map. Curves in the Connecticut River are the features noted on days of low visibility; the rather abrupt hills which occur in the Northampton to Greenfield section of the valley offer a number of easily identified points for distances up to fifteen miles; the range of hills beyond Greenfield yield possible but less surely identifiable points to over thirty miles. The valley has little manufacturing and no dust or smoke haze is noticeable over the towns.

Photographs were taken on Wratten and Wainwright panchromatic plates and on Type IR infrared plates, each with an A filter, No. 25, red. Visibility with the former is practically identical with that of the eye.

The results are given graphically. The slope of the curve changes steadily from 1.1 for days of low visibility to 1.7 for the clearest days. The agreement between points for hazy days and for days of continuous or intermittent rain, is striking.

From two of the formulas and Fig. 1 of the article referred to, rather interesting conclusions may be drawn, in connection with the value of the slope. The intensity of light of a certain wave-length is given by the formula

$$i = i_0 e^{-(\alpha+\beta+\gamma)x}, \tag{1}$$

in which i_0 is the original intensity of the light, and i its intensity after passing through the distance x, α and β are scattering and absorption coefficients due to air molecules, and γ the attenuation due to foreign particles, chiefly water drops and dust. It has been shown that for particles of index of refraction 1.33,

$$\gamma = \pi r^2 n k, \tag{2}$$

in which r is the radius of the particles, n their number per cc, and k a function of λ/r. This

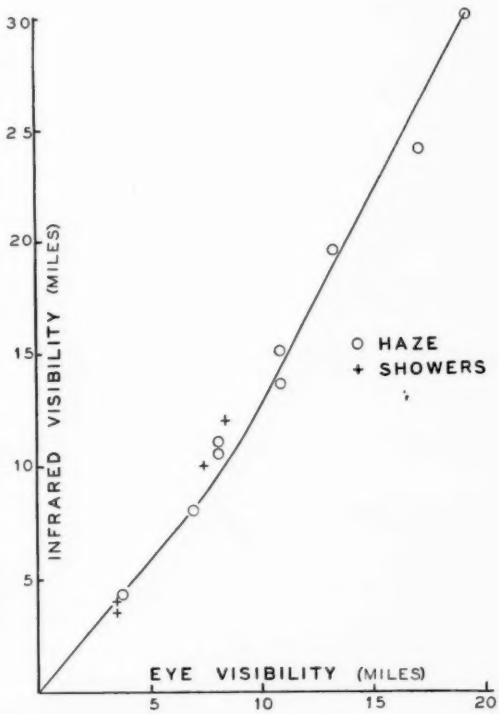

FIG. 1.

[1] E. O. Hulburt, "Attenuation of Light in the Lower Atmosphere," J. O. S. A. **25**, 125 (1935).

function is given graphically in the figure referred to.

It follows from Eqs. (1) and (2), that if α and β are small with respect to γ, which is true for wave-lengths longer than 300 mμ, two photographs taken under the same conditions except for the effective wave-lengths, will show visibilities inversely proportional to $\Sigma\gamma$; if one size of drop is chiefly responsible for the effect the ranges will be inversely proportional to the k's also.

The ratio of visibilities runs from 1.7 for clear days, through 1.4 for hazy days, to 1.1. The ratio of wave-lengths for the two plates is about 6 to 8, as the filter cuts at 580 mμ and the panchromatic plate is quite insensitive beyond 650 mμ, and the R sensitivity is chiefly from 740 to 840 mμ. Two parts of the k curve show decreasing k with increasing λ/r. For $\lambda/r<1$ the slope yields a value for k_1/k_2 of about 1.5 for a wave-length shift in the ratio of 1 to 1.3 (r assumed constant). A shift to somewhat greater or smaller values of λ/r decreases the ratio. For values greater than 1, the ratios of k increase from 1.1 for λ/r about 1 to 1.7 for $2<\lambda/r<5$, for the same change in wave-length. These correspond to values of r from 0.1 to 0.3μ for thin haze, to 0.6μ for thick haze, with photographic conditions duplicated in the middle range for particles of 3μ radius. The presence of larger particles with the smaller changes the $\Sigma\gamma$ values and may degrade the slope, but the upper limit of 0.3μ for the particles in thin haze seems definite. Hence thickening haze means a change in r as well as in n, with a slight change in size for most of the drops, or with an addition of comparatively few much larger drops.

Proceedings of the Twentieth Annual Meeting of the Optical Society of America

THE FRANKLIN INSTITUTE, PHILADELPHIA, PENNSYLVANIA
October 24–26, 1935

THE twentieth annual meeting of the Optical Society of America was held at the Franklin Institute, Philadelphia, Pa., October 24–26, 1935. The meeting was opened Thursday, October 24, at 9:30 A.M. with a brief address of welcome by Mr. Henry Butler Allen, Director of the Museum.

INVITED PAPERS ON THE MICROSCOPE AND MICROSCOPY

The sessions Thursday morning and afternoon, October 24, were devoted to papers on "The Microscope and Microscopy." The following papers were presented by invitation:

9:30 A.M.

Geometrical Optics of the Microscope, by W. B. Rayton, Bausch & Lomb Optical Company

Theory of Microscope Vision, by M. Herzberger, Eastman Kodak Company

Mechanical Design of the Microscope, by Henry F. Kurtz, Bausch & Lomb Optical Company

1:00 P.M.

Illumination and Manipulation of the Microscope, by H. W. Zieler, E. Leitz, Incorporated

Polarizing Microscopes, by L. C. Graton, Harvard University

Dark Field and Ultra Microscopes, by Simon H. Gage, Cornell University

Electron Microscopes, by C. J. Davisson, Bell Telephone Laboratories

MEETING WITH JOINT PHILADELPHIA PHYSICS SYMPOSIUM

On Thursday evening at 8:15 P.M. the Optical Society met with the Joint Philadelphia Physics Symposium and heard invited papers on "The Physical Basis of Sensation." The following papers were presented by invitation:

The Messages from Sense Organs, by Detlev Wulf Bronk, Johnson Foundation for Medical Physics

Electrical Studies of Vision, by H. Keffer Hartline, Johnson Foundation for Medical Physics

ANNUAL DINNER

At the Annual Dinner of the Society on Friday evening the Frederic Ives Medal for 1935 was awarded to Dr. George Ellery Hale, Honorary Director of the Mount Wilson Observatory of the Carnegie Institution. Acceptance of the medal on behalf of Dr. Hale was made by Dr. Charles G. Abbot, Secretary of the Smithsonian Institution.

LECTURE AND DEMONSTRATION AT THE FELS PLANETARIUM

After the dinner the members and guests attended a lecture and demonstration at the Fels Planetarium. Dr. James Stokley, Associate Director of the Museum, in charge of the planetarium, explained the mechanism of the instrument, giving an extended demonstration of its possibilities for representing astronomical phenomena.

MUSEUM OF THE FRANKLIN INSTITUTE

The opportunity to see and examine the interesting exhibits in the Museum was enjoyed by all.

BUSINESS SESSION

A brief business session of the Society was held Saturday morning at 9:00 A.M., President Rayton presiding. In the absence of the Secretary, K. S. Gibson acted as Secretary *pro-tem*.

The formal reports of the Secretary, Editor of Publications, and Treasurer, for the calendar year 1934* were laid before the Society. The reports were supplemented by informal reports of the changes since January 1, 1935. The reports were accepted.

The efforts of the Local Committee were acknowledged and appreciation was expressed for their efficient handling of the various details of the meeting.

REGISTRATION

Members 69, Guests 31, Total 100.

* Published in the J. O. S. A. **25**, 196 and 224 (1935).

Sessions for the Reading and Discussion of Contributed Papers

Sessions for the reading and discussion of contributed papers were held as follows:

Friday, October 25, at 9:30 A.M.
Friday, October 25, at 1:00 P.M.
Saturday, October 26, at 9:30 A.M.
Saturday, October 26, at 1:00 P.M.

The authors' abstracts of these papers are appended.

Meeting of the Board of Directors

Sessions of the Board of Directors were held on Wednesday, October 23.

K. S. Gibson, *Secretary Pro-Tem.*

Minutes of the Sixth Meeting of the Directors of the Optical Society of America, Incorporated

Philadelphia, Pennsylvania, October 23, 1935

Besides the transaction of routine business, the following items of general interest to members of the Society were taken up.

Announcements by Secretary

Results of letter ballots:

1. Appointment of representative and alternate on the American Standards Association Sectional Committee on Safety Glass, as follows:

E. D. Tillyer
H. F. Kurtz, alternate

2. Appointment of representative on the American Standards Association Motion Picture Sectional Committee, as follows:

A. C. Hardy

3. Reappointment of P. I. Wold as representative in the American Association for the Advancement of Science.

4. Reappointment of C. L. Bausch as representative on Division of Physical Sciences, National Research Council.

5. Appointment of representatives on Inter-Society Color Council, as follows:

L. A. Jones, *Chairman*

A. Ames, Jr.	A. E. O. Munsell
I. H. Godlove	A. H. Pfund
Deane B. Judd	W. B. VanArsdel

6. Holding of annual meeting at Franklin Institute approved.

7. Appointment of tellers to make count of ballots for 1935 election of officers and directors, as follows:

Fred E. Wright, *Chairman*

Dorothy Nickerson	Arthur Q. Tool

8. Appointment of Committee on Qualifications for Membership, as follows:

W. E. Forsythe, *Chairman*

F. L. Mohler	J. S. Friedenwald
A. H. Pfund	L. O. Grondahl

9. Ballot approving recommendation of Ives Medal Committee that the medal be awarded to Dr. George Ellery Hale.

10. Ballot appointing the following as voting delegates on Inter-Society Color Council:

L. A. Jones
D. B. Judd
W. B. VanArsdel

Membership

The secretary's report showed that since the last meeting of the Directors 15 new members had been elected as of January 1, 1935 and 10 new members as of January 1, 1936.

The secretary reported four deaths among the membership since the February meeting. In this connection it was voted "that in January of each year a portion of the *Journal of the Optical Society of America* be devoted to obituary notices for the past year under the head of 'Necrology'."

Transfer from Associate to Regular Membership

The following were transferred from associate to regular membership:

Stuart Campbell	(984 R)	H. B. Maris	(986 R)
Alan A. Cook	(957 R)	Harold Mestre	(624 R)
Myrl N. Davis	(959 R)	Sidney Newhall	(933 R)
H. J. Eckweiler	(860 R)	R. D. Nutting	(967 R)
Max Herzberger	(1006 R)	Leigh Page	(998 R)
R. S. Hunter	(947 R)	W. W. Stifler	(799 R)
H. D. Judd	(68 R)	H. T. Wensel	(981 R)
A. B. McLay	(993 R)	H. W. Zieler	(977 R)

Financial Status of the Society

The treasurer gave an informal report which showed that the Society had maintained its excellent financial condition.

Joint Meeting of Member Societies of the American Institute of Physics

The question of a joint meeting of all member societies of the American Institute of Physics was taken up and it was voted "that the Board of Directors favor a joint meeting with the member societies of the American Institute of Physics for next fall."

Change in Policy in Handling The Review of Scientific Instruments

The change in policy of the American Institute of Physics in handling *The Review of Scientific Instruments* was discussed and the following resolution was passed:

Resolved that for 1936 the Optical Society of America purchase for each of its members entitled to receive its Journal one subscription to *The Review of Scientific Instruments.*

Appointment of Associate Editors of the Journal

The following associate editors of the *Journal of the Optical Society of America* were appointed for a term of six years beginning January 1, 1936:

W. G. Cady
E. O. Hulburt
H. E. Ives
D. B. Judd
Leigh Page
A. H. Pfund

Report of Committee on Index

F. K. Richtmyer, chairman of the Committee on Index, expressed the hope to have the index published by mid-winter. It was voted "that the cumulative index cover the publication of the *Journal* through the year 1935."

Election of Officers

The count of the official tellers showed that the following officers and directors had been elected:

For President —A. C. Hardy
For Vice President—R. C. Gibbs

both for a term of two years beginning October 26, 1935; and for Directors, for a term of four years beginning October 26, 1935:

K. S. Gibson
G. R. Harrison

Committee to Consider Resolutions Supporting Science Advisory Board

The Committee to consider resolutions supporting the Science Advisory Board was discharged.

Report of Representative at Conference on Standards for Inspection of Motor Vehicles

An informal report was made by G. B. Pegram, representative of the Society at the conference on Standards for the Inspection of Motor Vehicles. He stated that at the conference it was decided to ask the American Standards Association to go ahead in the usual manner to provide standards for inspection of motor vehicles.

K. S. Gibson, *Secretary Pro-Tem.*

Program of Sessions for Contributed Papers*

(TITLES AND ABSTRACTS OF PAPERS)

1. A Polarizing Vertical Illuminator. LEON V. FOSTER, *Bausch and Lomb Optical Company.*

The normal form of vertical illuminator consists of a thin plate of glass inclined 45 degrees to the axis of the microscope objective and is used to reflect light through the latter on to opaque specimens. This illuminator is very inefficient. A vertical illuminator having about 5 times the efficiency of a plain glass illuminator can be made from calcite. Such an illuminator allows the direction of plane polarized light on the specimen and light reflected by the specimen passes through the illuminator and follows the ordinary ray refraction laws. The application of this illuminator to the examination of metal specimens and an instrument in which it is used will be described.

The present paper will appear in full in J. O. S. A.

2. Optics of Low Power Photomicrography. R. T. LOVELAND, *Eastman Kodak Company.* (Introduced by Brian O'Brien.)

Regular photographic lenses, when reversed in the holder, not only can be utilized for photomicrography but above the lowest power are more suitable than the special microphotographic objectives both because the object is nearer to the focal plane and because higher apertures are available. Considering the aperture requirements for resolution it is shown that the maximum apertures of F 4.5 of commercial microphotographic objectives needlessly limit the use of the simple microscope with its advantage of a large field. The optics of the various methods of illumination are discussed systematically. By first considering the case where no object condenser is used the field size aperture relationship and its dependence on the lamp is evident. The vignetting effect of the exit pupil is discussed. When an object condenser is introduced, the field size aperture relationship remains unchanged but the exit pupil can be controlled. The effect of the position and power of the object condenser is described. Possible arrangements are compared and one recommended.

3. The Measurement of the Aberrations of a Microscope Objective. R. KINGSLAKE, *Institute of Applied Optics, University of Rochester.*

A critical survey is made of the methods that have been employed for measuring the spherical aberration of microscope objectives. A new method somewhat resembling those suggested by Chalmers[1] and Cotton[2] for telescope objectives is described, which avoids some of the disadvantages of the others. In this method a thin metal plate having a small hole in it is moved by measured amounts across the tube behind the objective, and the point at which the narrow beam of light thus isolated intersects the image plane is located by suitable means. This gives a direct measure of the transverse aberration of that zone of the lens which contains the sliding hole, from which either the optical path difference or the ordinary longitudinal aberration of the zone can be readily computed. The limitations and precision of the method are discussed.

In order to plot the usual curve connecting aberration with aperture, it is necessary to express the aperture of the zone in question in some definite and significant manner. Some typical curves are given which indicate the possibilities of the method.

The Foucault knife-edge test was found to be readily adaptable to the measurement of the chromatic aberration of a microscope objective; typical curves for this test are also given for a number of objectives.

[1] Proc. Opt. Convention 1912, 156.
[2] Physica **1**, 274 (1921); Rev. d'optique **3**, 401 (1924).

4. The Ultra-Microscope in Motion Picture. R. A. WETZEL, GEORGE NEEDHAM AND HENRY ROGER, *College of the City of New York and Rolab Photo-Science Laboratory, Sandy Hook, Conn.*

Description of equipment used and a screen demonstration of the Brownian movement in colloidal solutions of copper, silver iodide and protargentum. On the latter we have a record of a new photo-effect the nature of which is as yet obscure.

5. Identification of Five Different Types of Gloss Effects. RICHARD S. HUNTER, *National Bureau of Standards.*

In the attempt to establish methods of measuring gloss, a number of different gloss effects have been encountered.[1] These effects have been classified into the five individual gloss types which are listed in Table I. In this table each gloss type is identified by the method by which it is perceived, by the type of materials on which it usually appears,

TABLE I. *Table of five gloss types.*

GLOSS TYPE	METHOD OF IDENTIFICATION	TYPES OF MATERIALS ON WHICH PERCEIVED	GLOSSMETERS APPLICABLE[3]
Objective gloss	Regular reflection	All materials from which brilliance of regularly reflected light is index of gloss	Hunter, Pfund, B & L Photoelectric for Paper
Subjective gloss	Contrasts of regular reflection and diffuse reflection	Papers, matt and semimatt coatings and white and light colored surfaces	Ingersoll, Pfund, Jones, Jones-Lowry Photoelectric
Sharpness of image	Sharpness of reflected images	Finishes, enamels, lacquers and all smooth image-reflecting surfaces	Hunter Gloss Comparator, Sward-Levy, Detroit Club, Wolff, Pfund (1925),
Bloom	Smear of semi-regular reflection accompanying regular reflection	Surfaces in which images and contrasts may be seen	Photographically recorded by Hunter Gloss Comparator
Smoothness	Absence of surface texture, irregularities, and blemishes. (Difficulty of recognizing the presence of the surface)	All objects, but particularly those which show a fair degree of regular reflection	Photographically recorded by Hunter Gloss Comparator

* The abstracts were preprinted in the program subject to correction by the authors. Corrections that have been communicated to the secretary have been made in the abstracts here reprinted.

and by the gloss measuring instruments which have been developed to measure it.

A general definition of gloss to cover all types of surfaces might read "The gloss of a sample is its ability to reflect light regularly (without scatter)." In order to establish gloss on a physical basis, preparations are being made to examine surfaces with the gonio-photometer[2] (reflection-distribution photometer) after they have been classified according to appearance effect. It is hoped that this work will establish numerically the different gloss-measuring scales which seem to be demanded by the study of appearance effects.

[1] R. S. Hunter, *The Measurement of Gloss*, J. O. S. A. **25**, 47 (1935).
[2] H. J. McNicholas, *Equipment for Measuring the Reflective and Transmissive Properties of Diffusing Media*, Bur. Standards J. Research **13**, 211 (1934) (RP 704).
[3] References to these gloss measuring instruments are given in the bibliography of the circular describing the Hunter Gloss Comparator, Scientific Section Circular No. 493 of the National Paint, Varnish and Lacquer Association, Washington, D. C., October 1935.

6. A Null Method Photoelectric Reflectometer. RICHARD S. HUNTER, *National Bureau of Standards.*

The Hunter reflectometer[1] employs a pattern-mirror photometer head[2] at the apex of a 45° right triangle and a lamp movable between two diffusely reflecting surfaces along the hypotenuse. A direct conversion of this visual reflectometer to a photoelectric instrument has been achieved by using two dry-disk photoelectric cells. Each of the cells is placed facing one of the reflecting surfaces along the line of sight used in a visual instrument. The two dry-disk cells are connected in series and a galvanometer is connected across the cells to register when one cell generates a current greater than the other. The single lamp in the instrument may be run from the ordinary electrical supply since the effects of small voltage variations cancel out in the cell circuit.

The same reflection scale used with the visual instrument is employed in the new device. The construction of this scale and requirements for its use are discussed. It happens that the spectral response of the Photox cell multiplied by the energy distribution of the tungsten gas-filled lamp gives a first approximation to the normal visibility curve. Further improvement of this response by means of selective filters is being tried. The precision of the instrument is high, settings to one-tenth of one percent being possible with a sensitive galvanometer. Good agreement has been obtained between reflection values of nearly neutral porcelains obtained with this instrument and values obtained visually.

[1] Scientific Section Circular No. 461 of National Paint, Varnish and Lacquer Assoc., Washington, D. C., April, 1934.
[2] R. S. Hunter, J. O. S. A. **24**, 54 (1934).

7. A Subtractive Colorimeter for the Measurement of Small Chromaticity Differences between Surfaces of Moderate Spectral Selectivity of Reflectance. DEANE B. JUDD, *National Bureau of Standards.*

Much routine colorimetric testing consists of the determination of the magnitude of small chromaticity departures from a standard color. A subtractive colorimeter for colors of plane and cylindrical surfaces has been built for such determinations. A projection lamp movable between the standard and comparison surfaces illuminates them perpendicularly. These surfaces are viewed at 45° and brought into optical juxtaposition by a Lummer-Brodhun cube bearing the usual double-trapezoid pattern. The sample size required to fill the 9°×13° photometric field may be varied from 10 to about 1-cm diameter by appropriate lenses. Except for these clear optical elements and a daylight filter in the ocular, the observer views the standard directly, but he views the comparison surface through two double wedges, one of greenish, the other of yellowish glass. The thickness of each wedge traversed by the comparison beam may be varied by the observer until a complete color match is possible.

The wedges are first adjusted to match the standard; then the test specimen is substituted and the wedges reset. If the test specimen is yellower or greener than the standard the thickness of one wedge or the other will have to be increased to produce a match; but if it is redder or bluer the thickness will have to be decreased. The differences in the wedge settings can be translated into differences in trilinear coordinates on any fundamental colorimetric system provided the spectral reflectance of the comparison surface is approximately known and the spectral transmissivities of the wedges are accurately known. Calibration is planned both in terms of the ICI standard system[1] and the uniform-chromaticity-scale system,[2] but it is likely that the latter will be used most because of its simpler connection to least perceptible chromaticity differences.

Because of the large, patterned field and high brightness, this instrument takes full advantage of the chromaticity sensibility of the observer. It is not applicable to highly selective samples because the resulting nearly homogeneous ight is too little changed in spectral distribution by passage through the wedges.

[1] Deane B. Judd, J. O. S. A. **23**, 359 (1933).
[2] Deane B. Judd, J. O. S. A. **25**, 24 (1935).

8. An Aid to the Calculation of Tristimulus Specifications. JOSEPH RAZEK, *University of Pennsylvania.*

The calculation of the tristimulus specifications of a sample whose spectral transmittance or reflectance is known is generally made by multiplying the spectral energy of the illuminant, E, the respective tristimulus values for the pure spectrum, $\bar{x}$, $\bar{y}$ and $\bar{z}$, and the spectral reflectance, R, (or transmittance) of the sample. The products, $\bar{x}ER$, $\bar{y}ER$ and $\bar{z}ER$, are determined at 10 millimicron intervals and the respective sums obtained. Tables of values of $\bar{x}E$, $\bar{y}E$ and $\bar{z}E$, which facilitate the calculations have been given by D. B. Judd.[1]

However, in using these tables over one hundred separate multiplications must be made for each sample. To eliminate the repeated multiplications, index cards were prepared on which were entered the values of $\bar{x}ER$, $\bar{y}ER$ and $-ER$. For each wave-length ninety-nine cards for values of R from 0.01 to 0.99 and nine additional cards of distinguishing color for values of R from 0.001 to 0.009 were required. Similar sets were prepared for each wave-length at 10 millimicron intervals throughout the visible spectrum, resulting in about 3400 cards, suitably tabbed and arranged in trays.

By starting with a list of the reflectances at 10 millimicron intervals, it is only necessary to insert a marker at two points in each wave-length set, one at the card corresponding to the first two figures in the reflectance and another at the card corresponding to the third figure in the reflectance. The values of $\bar{x}ER$, $\bar{y}ER$ and $\bar{z}ER$ at the marked places are successively entered upon a printing adding machine and the three sums obtained, eliminating the multiplications and the transcribing of the separate values. The strip from the adding machine provides the permanent record for checking.

The advantages of this procedure are: (1) Ordinary printing adding machines are more readily available than calculating machines; (2) considerably less fatigue results from merely adding a series of numbers than from making a large number of multiplications, and transcribing the results; and (3) the time required for a calculation is reduced from about one hour to approximately 15 minutes. The disadvantages of the method are that at each wave-length two numbers instead of one are added for three significant figures in the reflectance, and the large amount of labor required in the original preparation of the cards is only justified where much work of this kind is to be done.

[1] D. B. Judd, J. O. S. A. **23**, 359 (1933).
[2] A. C. Hardy, J. O. S. A. **25**, 310 (1935).

9. Standardization and Specification of Railway Signal Colors. KASSON S. GIBSON, GERALDINE WALKER HAUPT AND HARRY J. KEEGAN, *National Bureau of Standards.*

A previous report[1] on this work was given at the Buck Hill Falls meeting in 1933. As a result of this cooperative work with the Signal Section of the Association of American Railroads and with Corning Glass Works, a tentative three-part specification[2] has been prepared for pressware and polished disks. The purpose of this specification is (A) "to define the minimum values of luminous transmission for signal glasses and the limits of chromaticity for signal colors produced by illuminant-glass combinations," (B) "to define the maximum deviations from the standard limit glasses, both in transmission and in chromaticity, to be permitted in glasses certified as duplicates of the standard limit glasses," and (C) "to provide for approved colored roundels, lenses, disks, and slides for railway signalling, to describe the materials and specify the appliances and appurtenances necessary to make the required tests and inspection." Part A of the specification is expressed in fundamental unambiguous terms and enables the bureau or any other standardizing laboratory to test for conformity thereto. Part B is expressed in the same terms as Part A and enables the bureau to test and certify duplicate limit glasses as required by Part C. Part C prescribes the method of testing signal glasses by the manufacturer or inspector who is equipped with a photometer and certified limit glasses.

The specification will be illustrated and discussed. Special acknowledgment is due Mr. J. C. Mock, Signal Engineer, Michigan Central Railroad, who has been in charge of this work for the A. A. R. Signal Section, and to Dr. H. P. Gage, Corning Glass Works, under whose supervision the large number of standard and duplicate limit glasses have been prepared.

[1] Gibson and Walker, *Standardization and Specification of Railway Signal Colors*, J. O. S. A. **24**, 57 (1934).
[2] *Signal Section Proceedings*, A. A. R. **32**, 441 (1935). (Minutes, Forty-first annual meeting.)
The present paper will appear in full in Bur. Standards J. Research.

10. Saturation as the Basis for Color Mixture in the Color-Blind. SELIG HECHT AND SIMON SHLAER, *Columbia University.*

Gauging the spectrum by the dichromat is incorrectly called color mixture; it is saturation, not hue, which determines the spectral distribution of the two mixture primaries.

We have gauged the spectrum of a protanope and a deuteranope with 458.7 and 570.0 mμ. Half the field presents mixtures of the primaries; the other half contains homogeneous light. The dichromat determines the spectral limits within which he can match each mixture merely by adjusting the brightness of the half-fields. The energy ratio 458.7/570.0 varies from 100 : 1 in the violet to 1 : 300 in the red, and is nearly 1:1 at the neutral point. Correspondingly the spectral matching range is nearly 50 mμ at the ends of the spectrum and 2 to 5 mμ between 475 and 535 mμ. The distribution of matching range is unsymmetrical about the neutral point, being longer on the red side.

Since the dichromat matches the neutral point with white light as well as with a specific mixture of two primaries, all mixtures of the primaries contain this neutral point mixture plus an excess of one primary. The spectrum to the dichromat then appears as one or the other primary plus white. Computation from mixture data gives the brightness ratio of the excess primary to the neutral point mixture. A plot shows the computed saturation as maximal at the ends and zero at the neutral point, the change being slow on the red side and fast on the blue side.

Actual saturation distribution measurements confirm the computations. Mixtures of white light plus a short-wave or a long-wave primary are in half the field, while spectral light is in the other, and the dichromat determines the spectral limits within which he can match a mixture merely by adjusting the brightness. The measured saturation distribution so nearly resembles the computed distribution as to leave little doubt that saturation is the basis on which the dichromat gauges the spectrum with two primaries.

The present paper will appear in full in J. General Physiology.

11. The Centers of Ocular Rotation and Other Characteristics of Vertical and Horizontal Eye Movements. WALTER R. MILES, *Yale University.*

The majority of investigators who have studied the center of ocular rotation have confined themselves to the horizontal plane probably because of certain technical difficulties involved in providing for both planes in the same piece of apparatus. The present study took advantage of a relatively simple photographic technique. Light reflected from a small area near the apex of the cornea

was recorded by an enlarging camera when the subject's eye moved in alternately fixating pairs of points subtending equal degrees of visual arc in vertical and horizontal planes. Vertical displacements of the apex of the cornea are approximately 0.8 as large as horizontal displacements thus indicating that the center of rotation for vertical movements is nearly 2 mm anterior to that for horizontal movements. Vertical movements have a slower angular velocity and terminate less promptly and less precisely than is typical for horizontal movements subtending similar linear distances. The relation of these results to certain phenomena of perception is considered. Slides are used.

12. Measurements on the Tolerance to Light in Non-photophobic Individuals and in Certain Pathological Conditions. CHARLES SHEARD, *Division of Biophysical Research, The Mayo Foundation.*

The ensemble used in these investigations is a simple type of glarometer carrying a set of test letters on a target fastened at the center of a ground glass screen which, in turn, through suitable regulation of the light transmitted by it, furnished the background field used for glare purposes. Numerous tests, the details of which will be presented, on 144 individuals who did not complain of intolerance to light under ordinary conditions, with a target illumination of 6.4 footcandles, showed that the weighted average value of field illumination necessary to produce glare-out was 240 footcandles for the left eye, 295 footcandles for the right eye and 380 footcandles with binocular observation of the target. Frequency distribution curves and tabulations will be presented. The significance of these data in the physiology of vision will be discussed. The placement of various types of absorptive glasses in front of the eyes tested did not alter the glare-out values. Investigations on the effects of change of illumination on the letters, serving as the test object, on the glare-out values showed that the ratio between glare-out and target illuminations was a constant. Within the error of experiment, the quality of illumination of the glare field has no effect on glare-out values. Data concerning measurements on tolerance to light in a group of 51 exophthalmic goiter cases, before and after operation, will be presented. In general, in such pathologic conditions, and before treatment or operation, the glare-out value is from forty to sixty percent of the average value obtained in normally light-sensitive individuals. Results subsequent to thyroidectomy will be given.

13. A Rational Visibility Meter. MATTHEW LUCKIESH AND FRANK K. MOSS, *Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland, Ohio.*

Notwithstanding the facts that our civilization is largely built upon seeing and that human safety and welfare depend upon the visibility of objects and visual tasks, measurements of visibility have not been available for specifying conditions for seeing. Therefore, the recent development of a portable instrument for obtaining such measurements is correspondingly important. The instrument provides a means for appraising the visibility of various objects and visual tasks and for specifying practicable levels of illumination upon a rational basis.

The visibility meter consists of two colorless filters with precise gradients of density which may be rotated simultaneously in front of the eyes while looking at an object or while performing a visual task. The observer holds the instrument before the eyes and with a finger of the right hand turns a disk which rotates the gradients until the visual threshold is reached. The photographic gradients not only reduce the apparent brightness of the visual field but also lower the contrast of objects with their backgrounds due to their diffusing characteristic. The latter greatly increases the range and sensibility of the instrument.

Some characteristics of the instrument are summarized as follows:

Scale of relative visibility
Range, 1 to 20 minutes visual angle. The threshold size of the object of regard determined by the criterion of resolution of critical details of the object. Unity on the scale represents the practical limit of resolution by average normal vision.
Calibrated for *resolution* of black objects upon a white background (also white on black) at a brightness of 10 millilamberts. Another calibration is based upon mere recognition of the *presence* or *absence* of an object.

Scale of recommended footcandles
Range, 1 to 1000 footcandles. Reading 8-point Bodoni type under various levels of illumination formed a series of visual tasks of various degrees of difficulty.
Calibrated by subjects possessing normal vision at a fixation-distance of 14 inches, and upon the assumption of 10 footcandles on the reading matter as a conservative standard of "ease of seeing."

A complete description of the design and use of this instrument, and particularly of its scientific foundations, is available elsewhere.

The present paper appeared in full in J. Frank. Inst. **220**, 4 (1935).

14. An Instrument for the Measurement of the Graininess of Photographic Materials. E. M. LOWRY, *Kodak Research Laboratories.*

The term "graininess" as applied in this paper refers to the subjective factor which has been well defined by Jones and Deisch[1] as: "the sensation or impression of nonuniformity in a photographic deposit, produced upon the consciousness of the observer when such a deposit is viewed." The physical aspect of the problem, that is, the size of the ultimate silver grains, the size frequency and the number of grains per given area, has been defined in this laboratory as "granularity" and in this paper the term will be so used.

Jones and Deisch used a method of measurement in which the observer determined the distance at which a magnified image of the deposit ceased to appear as nonhomogeneous. This is essentially the same as measuring the necessary magnification when the viewing distance is kept constant. It is the latter method which has been applied in the design of the instrument to be described.

A number of methods for the measurement of graininess have been proposed at various times; among these have been the so-called physical methods such as the measurement of Callier's "Q" factor or the scanning of the silver deposit and recording its nonhomogeneity by means of the response of a photoelectric cell or other light sensitive

device. These methods measure the granularity of the deposit rather than its graininess. In the last analysis the point of chief interest is the effect upon the consciousness of the observer, and so far as present knowledge is concerned a subjective method is the only one which gives promise of direct interpretation.

Following the description of the apparatus and the defining equation some results showing the relation between field brightness and graininess are given for a range of brightnesses from 6 to 100 millilamberts. Data showing the effect of constant field brightness upon the graininess of several different emulsions are presented and a proposal for applying the results to the problem of graininess measurement is made. Results are also included on "Q" factor determinations as a measurement of graininess, together with a description of the apparatus used for this investigation.

[1] L. A. Jones, and N. Deisch, *The Measurement of Graininess in Photographic Deposits*, J. Frank. Inst. **190**, 657 (1920).

The present paper appeared in full in J. O. S. A. **26**, 65 (1936).

15. Minimum Useful Gradient as a Criterion of Photographic Speed. LOYD A. JONES AND M. E. RUSSELL, *Kodak Research Laboratories.*

In order to express numerically the speed of a photographic material it is necessary to choose a criterion upon which the computation may be based. Two criteria have been used quite generally: (a) The inertia, and (b) the exposure required to produce a specified (usually low) density. From both theoretical and practical considerations neither of these criteria is entirely satisfactory.

The present paper discusses the possibility of using a minimum limiting gradient as a criterion of practical effective speeds, at least in the amateur field. This is not a new proposal but has not been used to any great extent because of the alleged difficulty of determining simply and precisely the exposure corresponding to a specified minimum gradient.

A differential photometer, gradient photometer, is described which, when used with a sensitometric strip of proper characteristics, makes the determination of the minimum limiting gradient extremely simple and yields results of high precision. Data are given illustrating the agreement between different observers and the order of precision obtainable.

It is not proposed to use a constant value of minimum gradient, but a value which is some function of the maximum gradient (gamma) to which the material is developed. No definite proposal as to what this function should be is made, but in order to illustrate the application of the method a tentative assumption is made that the criterion to be used in computing a value of practical speed is the exposure required to produce a gradient which is equal to one-half of maximum gradient (gamma).

The superiority of this criterion over those in common use is discussed at some length. Illustrative data obtained by examination of some fifty different amateur negative materials are given which show the relation between speed numbers determined in this manner and those determined by the "inertia" and "fixed density" methods.

The paper closes with suggested specifications for a sensitometer, gradient photometer, and technic for the standardization of this method.

The present paper will appear in full in J. O. S. A.

16. The Summation of Different Color Radiations by a Photographic Emulsion. J. H. WEBB, *Kodak Research Laboratories.*

The problem of determining the manner in which a photographic emulsion sums different color radiations divides itself naturally into two parts depending upon whether the different radiations are applied simultaneously or successively. The first part of this problem was thoroughly investigated by A. Van Kreveld who announced a beautifully simple theoretical law for the manner in which an emulsion adds different colors applied simultaneously. In the first part of the present paper results are presented in verification of Van Kreveld's summation law. The second, and main, part of this paper is concerned with the addition of colors applied successively. A special technic had to be developed for the execution of these experiments and for the reduction of the data. The method of making the exposures consisted of crossing intensity scale curves, and the comparison of the separate and combination color exposures was carried out by graphical means. The results show that an emulsion adds radiations of different color (applied simultaneously or successively) according to the same rule that it adds radiations of identical quality. Van Kreveld's summation law was tested for the case of successive addition and was shown to hold in a restricted form here also. A brief discussion of the theoretical significance of the results is included.

The present paper appeared in full in J. O. S. A. **26**, 12 (1936).

17. A Spurious Photographic Reciprocity Defect for Alpha-Rays. ROBERT N. WOLFE AND T. RUSSELL WILKINS, *University of Rochester.*

The study of the validity of the photographic reciprocity law for alpha-rays has been extended from a ratio of source intensities of 7 to 1 previously reported in the literature to 171 to 1. The source intensities were equivalent to 1.38 millicuries/cm^2 and 8.06×10^{-3} millicuries/cm^2, respectively (the strongest polonium source of which we have found a record being 39.1 millicuries/cm^2 reported by Erbacher in 1933). There is no true failure of the reciprocity law for the process emulsion used, but an apparent failure has been found due to the growth of latent image. However, if the short exposure to the strong source is made in the middle of the long exposure to the weaker source, this effect is balanced out. To avoid an apparent reciprocity failure due to aggregate recoil the sources were coated with silver to a diffuse optical density of 0.50 corresponding to a film thickness of 0.1μ.

18. The Temperature Coefficient of Photographic Sensitivity. Part II. Effect of Low Temperature on the Photographic Action of α-Particles. S. E. SHEPPARD, T. RUSSELL WILKINS, E. P. WIGHTMAN AND R. N. WOLFE, *University of Rochester and Eastman Kodak Company.*

The Kinoshita exponential equation for the relation of the density of the developed film to exposure to α-rays has

been found not to apply for a diffuse source of the α-radiation. But it has been found possible to fit the experimental data for both a Process and a doubly coated Panchromatic emulsion with a combination of two somewhat similar expressions, namely, (1) $G=G_m(1-e^{-k(N/G_m)})$, in which G and G_m are, respectively, the number of grains already hit and the total number, under unit area of emulsion surface, and N is the number of α-rays falling on that unit area during the time of exposure, and (2) $D=1.57D_m(1-e^{-(G/G_m)})$, in which D and D_m are, respectively, any density (corrected for fog) and the maximum density, for complete exposure and infinite development time.

At room temperature with complete penetration of the emulsion by the α-particles, k/G_m is approximately equal to the projective area of the average sized grain. At -185°C the value of k/G_m is less than this. The relationship of D/D_m to N (at room temperature) has been established for various values of k/G_m. The larger k/G_m the steeper is the initial rise in the curves expressing this relationship.

With a double coated emulsion the equations are applied to each coating separately and the corresponding calculated D's are added. Assuming the relative thickness of the upper and lower layer in the Panchromatic emulsion to be approximately equal the resulting composite theoretical curve fits the experimental data surprisingly well; and the size of k/G_m closely approximates the measured values for each layer.

It has been shown that at room temperature α-rays produce about the same relative effect as light on the process and on the Panchromatic emulsions examined, with respect to each other, and that when the exposure to α-rays is made at a very low temperature there is a loss of contrast similar to that found with light (using an intensity scale) but that this loss cannot be attributed either entirely or in part to a reciprocity law failure as in the case of light. There is considerably less loss in H. and D. speed, however, than there was with light.

An apparent reciprocity law failure was observed for α-ray exposures and was examined in detail separately. The results will be reported in another paper.

Photomicrographic studies reveal that for the same exposure there are less grains developed when exposure is made at low than at ordinary room temperature and that the rate of development in the earlier stages is less for the emulsion exposed at low temperature. This last is also shown in the gamma-time of development curves.

An enormous effect has been observed on speed and contrast, when exposing at low temperature films given oxidation treatment followed by replacement of the soluble halides removed by the said treatment. This has not yet been explained.

The hypothesis previously proposed that at least part of the loss in contrast, and of speed, where there is such loss, by exposure at low temperature (-185°) is caused by the immobility and nonabsorption by halogen acceptors at the time of the exposure, which bromine as the temperature rises can then attack the latent image, appears to hold for these experiments as it did for light exposures. This assumed attack by the bromine is believed to cause a slowing up of the rate of development, which is substantiated by photomicrographic studies.

It should be stressed that for investigations with α-particles and other similar types of radiation, especially with diffuse sources, the process emulsion is much more suitable than others containing flat grains and multiple layers of emulsion, since the number of complicating factors is at a minimum.

19. The Second Law of Thermodynamics as Applied to Optics. ENOS E. WITMER, *University of Pennsylvania.* (Introduced by Joseph Razek.)

Every elementary beam or pencil of light of a given frequency, ν, and plane of polarization may be assigned a temperature which is determined by its brightness, K_ν. This temperature may be calculated by means of the formula[1]

$$K_\nu = h\nu^3 n^2/c^2(e^{h\nu/kT}-1),$$

which is essentially the Planck radiation formula.

Here n is the index of refraction of the medium in which the pencil of light occurs; the other symbols have their usual meanings. According to the second law of thermodynamics, no naturally occurring process will carry energy from a lower temperature to a higher temperature. This leads to the interesting and important conclusion that no optical device can be constructed that will increase the brightness of a beam of light, provided that the light ends up in a medium of the same refractive index as that in which it started. (More generally $(1/n^2)K_\nu$ cannot increase.) For example, no combination of mirrors and lenses will enable one to produce a beam of the brightness of an arc light from the light emitted by sources of lower temperature such as incandescent lights. It is possible, of course, to demonstrate this result from the laws of geometrical optics; nevertheless, it is interesting to note that it follows very simply from such a general law as the second law of thermodynamics.

[1] *Handbuch der Experimentalphysik*, Vol. IX, Part 1, pp. 373–374. Planck and Masius, *The Theory of Heat Radiation*, pp. 15, 176.

20. Decay of Luminescence in Solid Solutions. D. H. KABAKJIAN, *University of Pennsylvania.* (Introduced by H. C. Richards.)

A number of chemical compounds known as solid solutions have the property of luminescing on absorbing energy from various sources. The energy may be absorbed in the form of visible or ultraviolet light, x-rays or rays from radioactive sources. A luminescent compound excited continuously by a constant source of energy will usually show a decay of luminescence in time, and various theories have been advanced to explain this decay. These theories have a direct bearing on the nature of the so-called active centers (Lenard centers) producing luminescence. Since a solid solution consists of a small number of molecules of a solute associated with a large number of solvent molecules, it has been assumed that the active centers consist of an aggregation of the molecules of the solvent around a molecule of the solute known as the "activator," and that the absorption of energy results in destroying these aggregates

with emission of light. This should result in an exponential decay of luminescence which does not agree with the experimental determinations of this property.

The author has suggested in a previous paper that luminescence, being due to displacement of electrons and their return to their normal orbits, is a reversible phenomenon and that the decay might be due to other reactions taking place simultaneously in the material but not necessarily connected with luminescence. The reactions suggested are chemical decomposition of the compound and the changes in the transmission coefficient of the material due to irradiation. It has now been possible to prepare luminescent zinc borate compounds in which these two factors can be practically eliminated. As a result these compounds show no appreciable decay in time even when continuously excited by rays from strong radioactive sources.

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21. Infrared Dispersion of Selected Liquids and Solids. A. H. PFUND, *Johns Hopkins University.*

Since Rayleigh's law of scattering by small particles is advantageously studied in the infrared, a knowledge of the refractive indices of the materials used is imperative. The solids must be available in finely-divided form and the liquids must be among the most transparent. Of the liquids CS_2 and CCl_4 were studied in hollow prisms equipped with windows of polished rocksalt. The curves for these liquids show, in the interval extending from 0.6μ to about 9μ, interesting types of anomalous dispersion. The solids: cerussite ($PbO-CO_2$), zincite (ZnO), rutile (TiO_2), barytes ($BaSO_4$) and sphalerite (ZnS) were, of necessity, investigated by the less accurate method involving the reflection of light from highly polished surfaces formed on the several crystals.

The present paper will appear in full in J. O. S. A.

22. The Dispersion of Several Metals in the Visible Region. H. M. O'BRYAN, *Harvard University.*

The optical constants of germanium, aluminum, manganese, cerium, lanthanum, and the alkaline earth group have been obtained from surfaces prepared by vacuum distillation and studied in the same high vacuum chamber. The angles of principal incidence and azimuth were measured by a modification of Brewster's method. One Nicol prism mounted in vacuum serves as both analyzer and polarizer, extinction being observed by means of a half-silvered mirror outside the vacuum chamber. The light first reflected from the metal surface was returned along its original path by a reflection at normal incidence from a second mirror. The second mirror was rotated on an arm about the axis of the metal mirror and through twice the angle by gears. The speed with which measurements could be taken made possible observations on many surfaces of each element and even during the process of evaporation.

All the alkaline earth metals have higher atomic rates of absorption and lower reflecting powers than the alkali metals. Magnesium is the most metallic of this group and beryllium almost nonmetallic. Germanium is relatively transparent to the extreme red and infrared and very similar to silicon in its optical properties. The reflecting power of aluminum at normal incidence as calculated from its refractive index and extinction coefficient is just above 90 percent for the entire visible region. Exposing any of these metals except germanium to the atmosphere caused a very great change in the observed principal angles.

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The present paper will appear in full in J. O. S. A.

23. A Simple Criterion for the Absence of Secondary Spectrum. FRED H. PERRIN, *Institute of Applied Optics, University of Rochester.*

It has been shown in the reference that, if Hartmann's formula holds for both glasses of a thin achromatic doublet, the condition for complete achromatism is that the values of λ_0 for both glasses should be the same. The present paper shows that, if λ_0 is plotted against ν, the common optical glasses fall near a single straight line that is inclined at a considerable angle to the axis of ν. Thus a large difference in ν, which is desirable, is associated with a large difference in λ_0. However, some materials, like fluorite, fall well outside this line and so can be used with an appropriate glass to make a combination that is completely achromatic. The suitability of a material is shown at a glance by the plot, and an appropriate mate can be quickly selected if it exists. Since λ_0 represents the location of an absorption band, it is suggested that the criterion here described may have a more fundamental significance.

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The present paper will appear in full in J. O. S. A.

24. Plane Polarization in Sunlight Reflected by Nonpolished Surfaces. F. E. WRIGHT, *Geophysical Laboratory, Carnegie Institution of Washington.*

In connection with the problem of ascertaining the nature of the surface materials of the moon many measurements have been made of the changes produced in sun's rays on reflection by nonpolished surfaces of rocks and other substances. In each case a certain amount of plane polarization is introduced, depending on the nature of the material, on the character and inclination of its surface, and on the angle between the incident and outgoing beams (phase angle). An incident ray of sunlight, impinging on a fracture surface or on a roughly ground flat surface or on a powder of a solid, is in part absorbed and later radiated as heat; in part it is reflected, either directly or after repeated refractions, by small crystal cleavage faces and by fracture facets; in part it is diffracted and scattered by the uneven surface. The outgoing beam is therefore not a simple beam such as is reflected by a polished piece of glass. The measurements on the percentage polarization contained in beams of sunlight reflected by rough surfaces prove that through a certain range of phase angles the plane of vibration of the polarized portion of the beam is in the plane of incidence (negative polarization of Lyot) and not normal thereto, as in the case of reflection by a flat polished surface of glass (positive polarization). The measurements were

made with the special eyepieces described in 1934.[1] Analysis of the data of measurement on sunlight reflected under these conditions throws light on the causes of the observed negative polarization. The results of the measurements on plane polarization have enabled us to determine more definitely than has heretofore been possible the nature of the materials exposed at the moon's surface.

[1] F. E. Wright, J. O. S. A. **24**, 206–216 (1934). In this paper the second eyepiece, Fig. 3, page 214, is represented, by mistake, as containing a compound Rochon prism rather than the correct compound Wollaston prism.

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The present paper will appear in full in J. O. S. A.

25. Mechanism of Deposition of Evaporated Metal Films Exhibiting Optical Resonance. T. A. RUSSELL AND BRIAN O'BRIEN, *Institute of Optics, University of Rochester.*

Optical resonance observed in very thin metal films has long been explained by a patch like character of the metallic deposit.[1] There is much evidence in support of this. Theories accounting for this type of deposition have been less satisfactory. These fall into two classes: (I) A probability of reflection of metal atoms which is different for uncovered portions of the target surface than for portions of the target already covered by a layer of the metal; (II) lateral motion of the metal atoms upon the target surface before permanent condensation occurs. A series of experiments using metals of high boiling point (Al, Ag, Cu, and Au) have been undertaken to determine which of these theories best accounts for granular deposition. A beam of metal atoms evaporated through an aperture into a spherical glass enclosure at room temperature shows no measurable reflection or reevaporation in residual gas pressures from 1.6×10^{-3} to 5×10^{-7} mm Hg, although at the higher gas pressures the metal deposit extends far beyond the geometrical prenumbra due to molecular scattering. Thus theory (I) is eliminated. Deposit in the geometrical shadow of a wire (Cockcroft)[2] and a reduced density at the apex of a film deposited through a *V* stencil (Estermann)[3] have been considered the principal evidence in support of theory (II). These effects can likewise be accounted for by molecular scattering. A careful repetition of Estermann's experiment has been undertaken using a silver stencil and silver as the evaporating metal. A lower density of the deposit near the tip has been observed, in agreement with Estermann, but is dependent upon the residual gas pressure. The shadow of the stencil is sharp within one micron, so that atom migration over greater distances than this does not occur. Thus the principal published evidence in support of theory (II) is inapplicable, although this theory may be accepted with the provision that migration cannot exceed one micron. Cooling the target with liquid air has no effect upon the optical properties of the deposit, and thus, presumably, upon the migration. This may be explained by assuming that the metal atoms reach target temperature only after migration is completed.

[1] Wood, *Physical Optics*, second edition, p. 635.
[2] Cockcroft, Proc. Roy. Soc. **A119**, 293 (1928).
[3] Estermann, Zeits. f. Physik **33**, 320 (1925).

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